

Question 7

Prove that

$$\lim_{x \rightarrow 0} \frac{\sin x}{\tan x} = 1.$$

Question 7 - Solution

Rewrite the expression using the identity

$$\tan x = \frac{\sin x}{\cos x}.$$

Substitute into the fraction:

$$\frac{\sin x}{\tan x} = \frac{\sin x}{\frac{\sin x}{\cos x}} = \cos x,$$

for all x where $\tan x$ is defined and $\sin x \neq 0$.

Now take the limit as $x \rightarrow 0$.

Since cosine is continuous at 0,

$$\lim_{x \rightarrow 0} \cos x = \cos 0 = 1.$$

Therefore,

$$\lim_{x \rightarrow 0} \frac{\sin x}{\tan x} = 1.$$

□