

Question 6

Prove that

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{x} = 0.$$

Question 6 - Solution

We begin by factoring the expression.

Rewrite the fraction as

$$\frac{\sin x - x}{x} = \frac{\sin x}{x} - 1.$$

Now take the limit as $x \rightarrow 0$.

Using the fundamental trigonometric limit,

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

Therefore,

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} - 1 \right) = 1 - 1 = 0.$$

Hence,

$$\boxed{\lim_{x \rightarrow 0} \frac{\sin x - x}{x} = 0}.$$