

Question 5

Prove that

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} = 2.$$

Question 5 - Solution

We begin by using a trigonometric identity.

Recall that

$$1 - \cos(2x) = 2 \sin^2 x.$$

Substitute this into the expression:

$$\frac{1 - \cos(2x)}{x^2} = \frac{2 \sin^2 x}{x^2}.$$

Rewrite the fraction:

$$\frac{2 \sin^2 x}{x^2} = 2 \left(\frac{\sin x}{x} \right)^2.$$

Now take limits as $x \rightarrow 0$.

Using the fundamental trigonometric limit,

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

Therefore,

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 = 1^2 = 1.$$

Multiply by 2:

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} = 2 \cdot 1 = 2.$$

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