

Question 4

Prove that

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1.$$

Question 4 - Solution

We rewrite the expression in terms of sine and cosine.

Recall that

$$\tan x = \frac{\sin x}{\cos x}.$$

Substitute this into the limit:

$$\frac{\tan x}{x} = \frac{\sin x}{x \cos x} = \left(\frac{\sin x}{x} \right) \left(\frac{1}{\cos x} \right).$$

Now take limits of each factor as $x \rightarrow 0$.

From the fundamental trigonometric limit,

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

Also, since cosine is continuous at 0,

$$\lim_{x \rightarrow 0} \cos x = \cos 0 = 1,$$

and therefore

$$\lim_{x \rightarrow 0} \frac{1}{\cos x} = 1.$$

By the product law for limits,

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \cdot 1 = 1.$$

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