

Question 3

Prove that

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = 3.$$

Question 3 - Solution

We begin by rewriting the expression so that the fundamental trigonometric limit appears.

Write

$$\frac{\sin(3x)}{x} = \frac{\sin(3x)}{3x} \cdot 3.$$

Now take limits as $x \rightarrow 0$.

Since $3x \rightarrow 0$ as $x \rightarrow 0$, the fundamental trigonometric limit gives

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} = 1.$$

Therefore,

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{x} = \left(\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \right) \cdot 3 = 1 \cdot 3 = 3.$$

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