

Question 8

Prove from first principles that if

$$f(x) = x^2,$$

then

$$f'(a) = 2a.$$

Question 8 - Solution

By definition of the derivative,

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Substitute $f(x) = x^2$:

$$f'(a) = \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h}.$$

Expand the square:

$$(a+h)^2 = a^2 + 2ah + h^2.$$

Substitute back:

$$f'(a) = \lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - a^2}{h}.$$

Simplify the numerator:

$$f'(a) = \lim_{h \rightarrow 0} \frac{2ah + h^2}{h}.$$

Factor out h :

$$f'(a) = \lim_{h \rightarrow 0} \frac{h(2a + h)}{h}.$$

Cancel h (for $h \neq 0$):

$$f'(a) = \lim_{h \rightarrow 0} (2a + h).$$

Now take the limit:

$$f'(a) = 2a.$$

Thus, using first principles,

$$\boxed{f'(a) = 2a}.$$