

Question 7

Prove that the derivative of a constant function is zero. That is, if

$$f(x) = c$$

for all x , where c is a constant, then

$$f'(a) = 0$$

for every real number a .

Question 7 - Solution

By definition of the derivative,

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Since $f(x) = c$ for all x , we have

$$f(a+h) = c \quad \text{and} \quad f(a) = c.$$

Substitute into the difference quotient:

$$f'(a) = \lim_{h \rightarrow 0} \frac{c - c}{h}.$$

Simplify the numerator:

$$f'(a) = \lim_{h \rightarrow 0} \frac{0}{h}.$$

Since $\frac{0}{h} = 0$ for all $h \neq 0$, the limit is

$$f'(a) = 0.$$

Therefore, the derivative of a constant function is zero:

$$\boxed{f'(a) = 0}.$$