

### Question 6

Assume that  $f$  is differentiable at  $x = a$  and that  $f'(a) = 0$ . Prove that

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0.$$

**Question 6 - Solution**

By the definition of the derivative,

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}.$$

We are given that

$$f'(a) = 0.$$

Thus,

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0.$$

To make this explicit using  $\varepsilon$ - $\delta$  language, let  $\varepsilon > 0$  be given.

Since the limit defining  $f'(a)$  exists and equals 0, there exists  $\delta > 0$  such that whenever

$$0 < |x - a| < \delta,$$

we have

$$\left| \frac{f(x) - f(a)}{x - a} - 0 \right| < \varepsilon.$$

That is,

$$\left| \frac{f(x) - f(a)}{x - a} \right| < \varepsilon.$$

This proves that

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = 0.$$