

Question 2

Assume that f is differentiable at $x = a$ and that c is a constant. Prove that

$$(cf)'(a) = c f'(a).$$

Question 2 - Solution

By definition of the derivative,

$$(cf)'(a) = \lim_{h \rightarrow 0} \frac{cf(a+h) - cf(a)}{h}.$$

Factor out the constant c from the numerator:

$$(cf)'(a) = \lim_{h \rightarrow 0} \frac{c(f(a+h) - f(a))}{h}.$$

Rewrite the fraction:

$$(cf)'(a) = c \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

Since f is differentiable at a , the limit exists and equals $f'(a)$. Therefore,

$$(cf)'(a) = c f'(a).$$