

Question 1

Assume that f and g are differentiable at $x = a$. Prove that

$$(f + g)'(a) = f'(a) + g'(a).$$

Question 1 - Solution

By definition of the derivative,

$$(f + g)'(a) = \lim_{h \rightarrow 0} \frac{(f + g)(a + h) - (f + g)(a)}{h}.$$

Expand the numerator:

$$(f + g)(a + h) - (f + g)(a) = (f(a + h) + g(a + h)) - (f(a) + g(a)).$$

Group terms:

$$= (f(a + h) - f(a)) + (g(a + h) - g(a)).$$

Substitute back into the limit:

$$(f + g)'(a) = \lim_{h \rightarrow 0} \left[\frac{f(a + h) - f(a)}{h} + \frac{g(a + h) - g(a)}{h} \right].$$

Use the limit law for sums:

$$(f + g)'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h} + \lim_{h \rightarrow 0} \frac{g(a + h) - g(a)}{h}.$$

Since f and g are differentiable at a , both limits exist and equal $f'(a)$ and $g'(a)$, respectively. Therefore,

$$(f + g)'(a) = f'(a) + g'(a).$$