

Question 6

Assume that $\lim_{x \rightarrow a} f(x) = L$. Prove that

$$\lim_{x \rightarrow a} (f(x) - c) = L - c,$$

where c is a constant.

Question 6 - Solution

Let $\varepsilon > 0$ be given. We must show that there exists $\delta > 0$ such that if

$$0 < |x - a| < \delta,$$

then

$$|(f(x) - c) - (L - c)| < \varepsilon.$$

Simplify the expression:

$$|(f(x) - c) - (L - c)| = |f(x) - L|.$$

Since $\lim_{x \rightarrow a} f(x) = L$, by the definition of the limit there exists $\delta > 0$ such that whenever

$$0 < |x - a| < \delta,$$

we have

$$|f(x) - L| < \varepsilon.$$

Thus, for this same choice of δ ,

$$|(f(x) - c) - (L - c)| < \varepsilon.$$

Therefore,

$$\lim_{x \rightarrow a} (f(x) - c) = L - c.$$