

Question 5

Assume that $\lim_{x \rightarrow a} f(x) = L$. Prove that

$$\lim_{x \rightarrow a} (f(x) + c) = L + c,$$

where c is a constant.

Question 5 - Solution

Let $\varepsilon > 0$ be given. We must show that there exists $\delta > 0$ such that whenever

$$0 < |x - a| < \delta,$$

we have

$$|(f(x) + c) - (L + c)| < \varepsilon.$$

Simplify the expression inside the absolute value:

$$|(f(x) + c) - (L + c)| = |f(x) - L|.$$

Since $\lim_{x \rightarrow a} f(x) = L$, by definition of the limit there exists $\delta > 0$ such that whenever

$$0 < |x - a| < \delta,$$

we have

$$|f(x) - L| < \varepsilon.$$

Therefore, for the same δ ,

$$|(f(x) + c) - (L + c)| < \varepsilon.$$

This proves that

$$\lim_{x \rightarrow a} (f(x) + c) = L + c.$$