

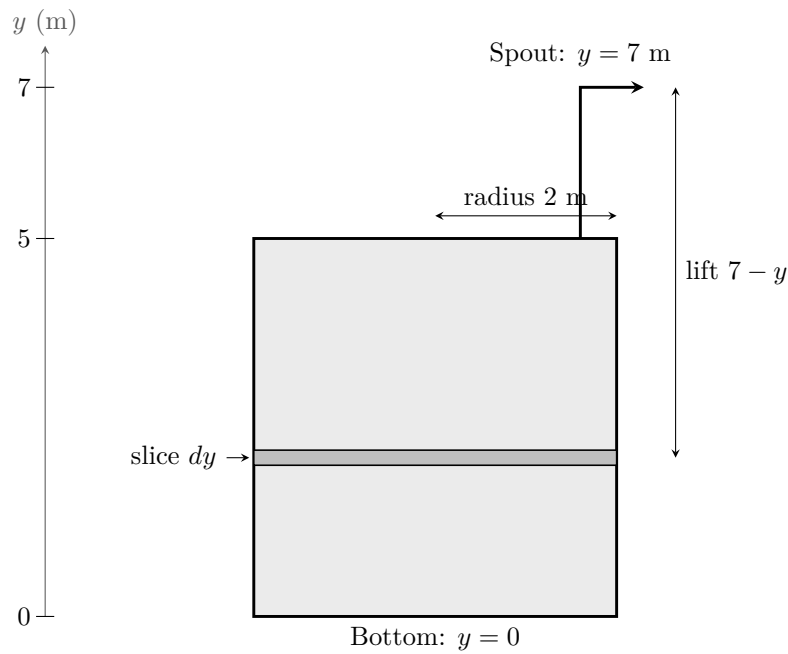
Question 9

A cylindrical tank of radius 2 m and height 5 m is full of oil. The oil has weight density

$$w(y) = 9000(1 + 0.1y) \text{ N/m}^3,$$

where y is the height (in meters) above the bottom of the tank.

Find the work required to pump all the oil to a spout located 2 m above the top of the tank.



Vertical section through the tank; the shaded band represents a horizontal slice.

Question 9 – Solution

1. Locate the slices and outlet.

Let y be height above the bottom. Oil occupies $0 \leq y \leq 5$, and the spout is at height $5 + 2 = 7$ m. The lift distance is $7 - y$.

2. Find the slice volume and weight.

The tank radius is 2 m, so

$$dV = \pi(2)^2 dy = 4\pi dy.$$

$$dF = w(y) dV = 9000(1 + 0.1y)(4\pi) dy = 36000\pi(1 + 0.1y) dy.$$

3. Set up the work integral.

Multiply each slice weight by its lifting distance:

$$W = 36000\pi \int_0^5 (1 + 0.1y)(7 - y) dy.$$

4. Expand and integrate exactly.

$$(1 + 0.1y)(7 - y) = 7 - \frac{3}{10}y - \frac{1}{10}y^2.$$

$$W = 36000\pi \left[7y - \frac{3}{20}y^2 - \frac{1}{30}y^3 \right]_0^5.$$

5. Apply the bounds and simplify.

$$\begin{aligned} W &= 36000\pi \left(35 - \frac{15}{4} - \frac{25}{6} \right) \\ &= 36000\pi \left(\frac{420 - 45 - 50}{12} \right) = \boxed{975000\pi \text{ J}}. \end{aligned}$$