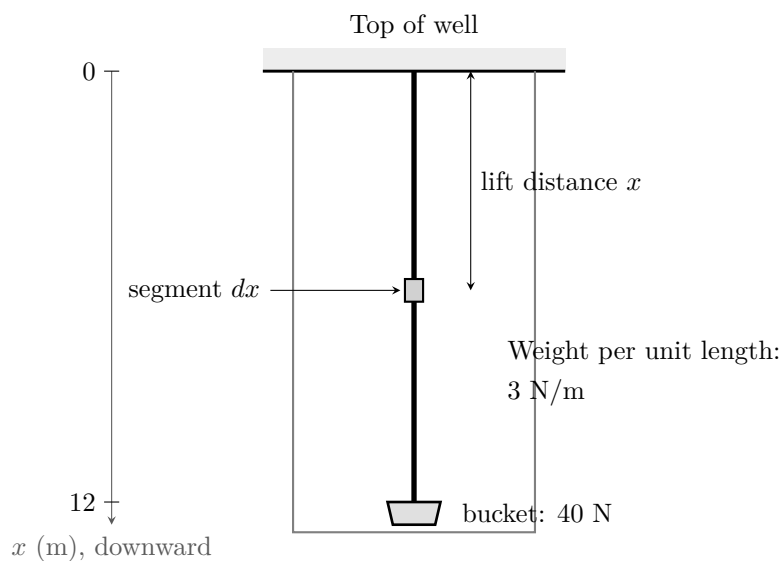


Question 8

A bucket is lifted from the bottom of a well that is 12 m deep. The bucket itself weighs 40 N. Attached to the bucket is a rope that weighs 3 N/m.

Find the work required to lift the bucket to the top of the well.



Initial configuration; x is measured down from the top.

Question 8 – Solution

1. Separate the bucket and rope contributions.

The bucket has constant weight, while different pieces of rope rise different distances:

$$W_{\text{total}} = W_{\text{bucket}} + W_{\text{rope}}.$$

2. Calculate the bucket work.

The 40 N bucket rises the full 12 m:

$$W_{\text{bucket}} = 40(12) = 480 \text{ J}.$$

3. Define a consistent coordinate for the rope.

Let x be the initial depth below the top of the well, with $0 \leq x \leq 12$. A segment of length dx weighs $3 dx$ newtons and rises x meters.

4. Integrate the rope work.

$$W_{\text{rope}} = \int_0^{12} 3x \, dx = \left[\frac{3}{2}x^2 \right]_0^{12} = \frac{3}{2}(144) = 216 \text{ J}.$$

5. Add and check the total.

$$W_{\text{total}} = 480 + 216 = \boxed{696 \text{ J}}.$$

As a check, the rope weighs $3(12) = 36$ N and its center of mass rises 6 m, giving $36(6) = 216$ J.