

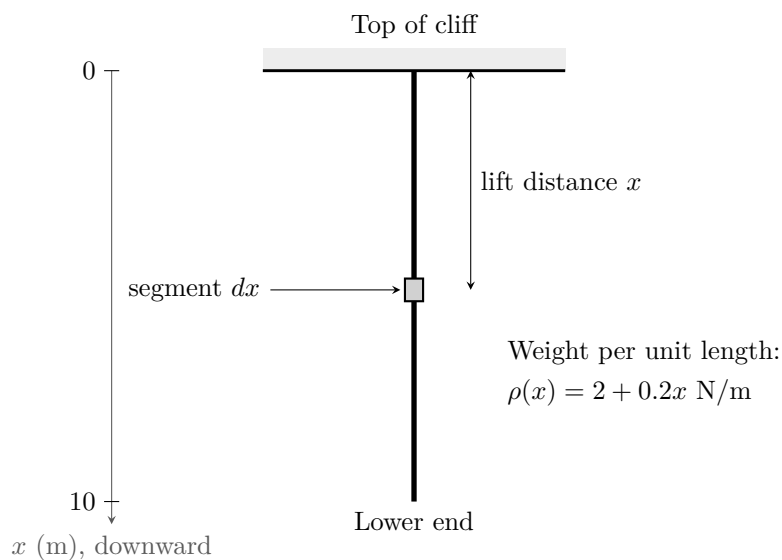
Question 7

A 10-meter rope hangs vertically from the edge of a cliff. The rope has variable linear weight density given by

$$\rho(x) = 2 + 0.2x \quad \text{N/m},$$

where x is the distance (in meters) measured downward from the top of the cliff.

Find the work required to pull the entire rope to the top of the cliff.



Initial configuration; x is measured down from the top.

Question 7 – Solution

1. Label each segment by its initial depth.

Let x be distance downward from the cliff top, as in the question. The rope occupies $0 \leq x \leq 10$ meters; $\rho(x)$ describes the initial weight distribution.

2. Find the segment weight.

For a short piece of length dx ,

$$dF = \rho(x) dx = (2 + 0.2x) dx.$$

The units are newtons because ρ is given in N/m.

3. Multiply by the distance lifted.

The segment at depth x is lifted x meters:

$$dW = x(2 + 0.2x) dx, \quad W = \int_0^{10} x(2 + 0.2x) dx.$$

4. Expand and integrate.

$$W = \int_0^{10} \left(2x + \frac{x^2}{5} \right) dx = \left[x^2 + \frac{x^3}{15} \right]_0^{10}.$$

5. Evaluate and state the work.

$$W = 100 + \frac{1000}{15} = 100 + \frac{200}{3} = \boxed{\frac{500}{3} \text{ J}} \approx 166.667 \text{ J}.$$