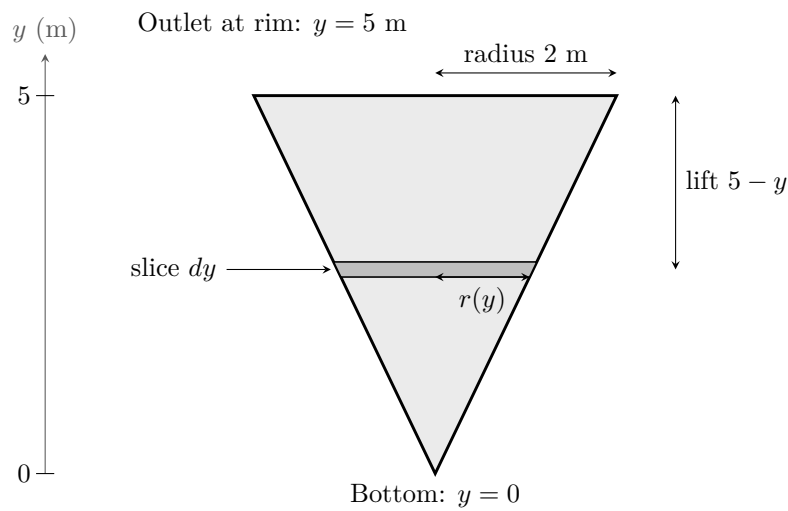


Question 6

A conical tank with its vertex at the bottom, height 5 m, and top radius 2 m is full of water. Water weighs 9800 N/m^3 .

Find the work required to pump all the water to the top of the tank.



Vertical section through the tank; the shaded band represents a horizontal slice.

Question 6 – Solution

1. Choose a height coordinate and determine the radius.

Measure y upward from the bottom vertex, so $0 \leq y \leq 5$. Similar triangles give

$$\frac{r(y)}{y} = \frac{2}{5} \implies r(y) = \frac{2y}{5}.$$

2. Find the volume and weight of a slice.

$$dV = \pi r(y)^2 dy = \frac{4\pi}{25} y^2 dy, \quad dF = 9800 dV = \frac{39200\pi}{25} y^2 dy.$$

3. Include the lifting distance.

The outlet is at $y = 5$, so a slice at y rises $5 - y$ meters:

$$W = \frac{39200\pi}{25} \int_0^5 y^2 (5 - y) dy.$$

4. Expand and integrate.

$$y^2(5 - y) = 5y^2 - y^3, \quad W = \frac{39200\pi}{25} \left[\frac{5}{3} y^3 - \frac{y^4}{4} \right]_0^5.$$

5. Apply the bounds and simplify.

$$\begin{aligned} W &= \frac{39200\pi}{25} \left(\frac{625}{3} - \frac{625}{4} \right) \\ &= \frac{39200\pi}{25} \cdot \frac{625}{12} = \boxed{\frac{245000\pi}{3} \text{ J}}. \end{aligned}$$