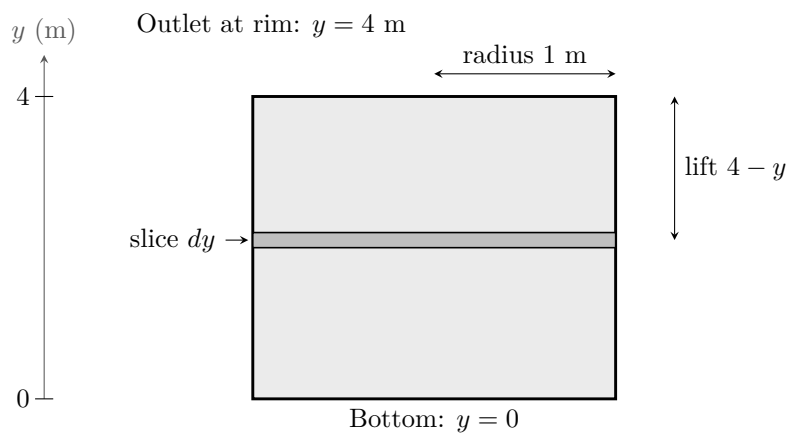


Question 5

A cylindrical tank of radius 1 m and height 4 m is full of water. Water weighs 9800 N/m^3 .

Find the work required to pump all the water to the top of the tank.



Vertical section through the tank; the shaded band represents a horizontal slice.

Question 5 – Solution

1. Choose horizontal slices.

Let y be height above the tank bottom, with $0 \leq y \leq 4$. A slice at height y has thickness dy .

2. Find the slice volume and weight.

The horizontal circular area is $A = \pi(1)^2 = \pi \text{ m}^2$. Hence

$$dV = \pi dy, \quad dF = 9800 dV = 9800\pi dy.$$

The density is already a weight density.

3. Find the distance to the outlet.

The outlet is at the top, $y = 4$, so the slice must rise $4 - y$ meters:

$$dW = 9800\pi(4 - y) dy.$$

4. Set up and evaluate the work integral.

$$\begin{aligned} W &= 9800\pi \int_0^4 (4 - y) dy = 9800\pi \left[4y - \frac{y^2}{2} \right]_0^4 \\ W &= 9800\pi(16 - 8) = 78400\pi. \end{aligned}$$

5. State and check the work.

$$\boxed{W = 78400\pi \text{ J}}.$$

The total water weight is $9800(4\pi) = 39200\pi \text{ N}$. Its average lifting distance is 2 m, giving $78400\pi \text{ J}$.