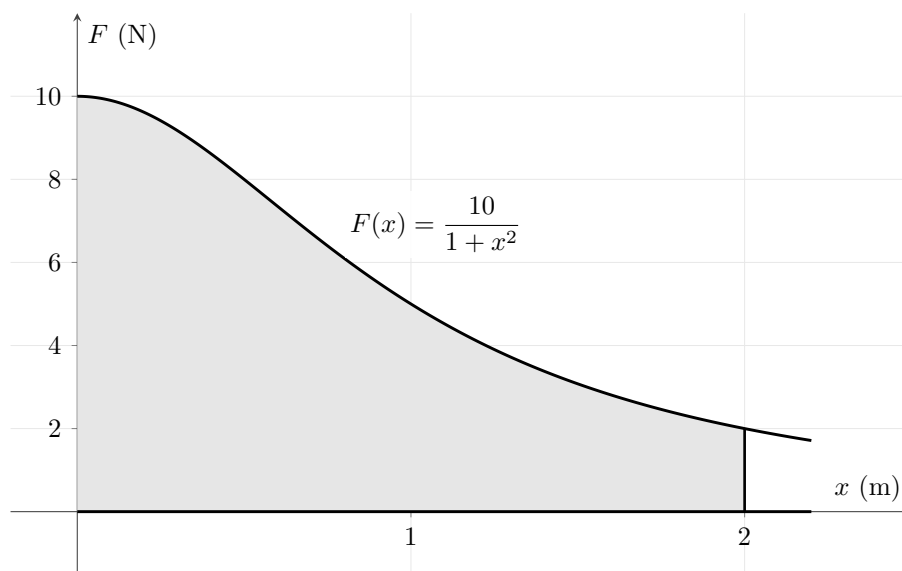


Question 3

A force acting on an object is given by

$$F(x) = \frac{10}{1+x^2} \quad (\text{newtons}),$$

where x is the distance (in meters) the object has moved along a straight line. Find the work done in moving the object from $x = 0$ to $x = 2$.



The shaded area represents work over the specified interval.

Question 3 – Solution

1. Identify the work model.

The force is positive over $0 \leq x \leq 2$, so the work equals the area under the force curve:

$$W = \int_0^2 F(x) dx.$$

2. Substitute the force.

$$W = \int_0^2 \frac{10}{1+x^2} dx = 10 \int_0^2 \frac{1}{1+x^2} dx.$$

3. Use the inverse-tangent antiderivative.

Since $\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$,

$$W = 10 [\arctan x]_0^2.$$

Inverse-trigonometric values are in radians.

4. Apply the endpoints.

$$W = 10(\arctan 2 - \arctan 0) = 10 \arctan 2.$$

5. State the exact and approximate work.

$$\boxed{W = 10 \arctan 2 \text{ J}} \approx 11.0715 \text{ J}.$$

The force decreases from 10 N to 2 N; the resulting work lies between $2(2) = 4 \text{ J}$ and $10(2) = 20 \text{ J}$.