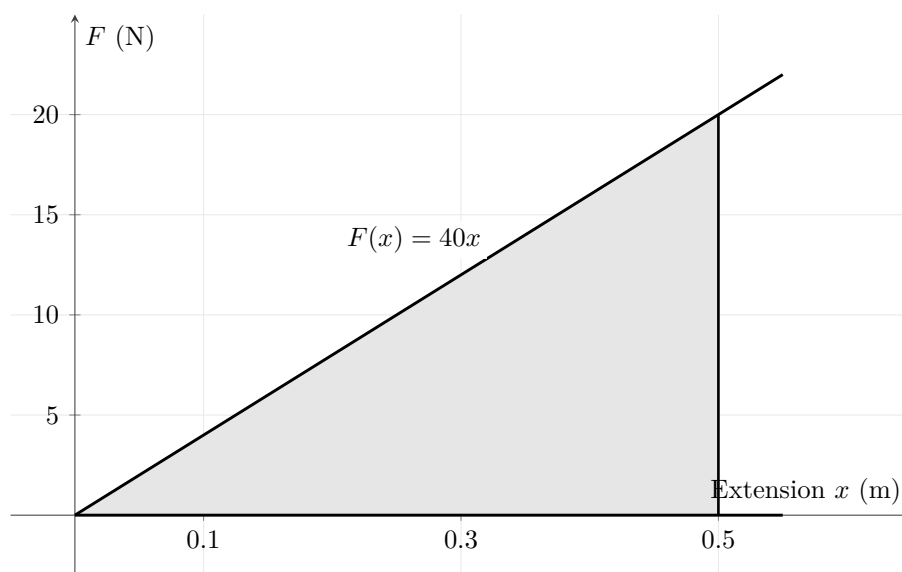


Question 2

A spring has natural length 0.5 m. A force of 12 N is required to stretch the spring to a length of 0.8 m.

Find the work required to stretch the spring from its natural length to a length of 1.0 m.



The shaded area represents work over the specified interval.

Question 2 – Solution

1. Use extension beyond natural length.

Let x be extension, not total spring length. The calibration force produces

$$x = 0.8 - 0.5 = 0.3 \text{ m.}$$

2. Find the spring constant.

Hooke's law for the required stretching force is $F = kx$:

$$12 = k(0.3) \implies k = \frac{12}{0.3} = 40 \text{ N/m.}$$

3. Translate the requested lengths into bounds.

The initial and final extensions are

$$x_0 = 0.5 - 0.5 = 0, \quad x_1 = 1.0 - 0.5 = 0.5 \text{ m.}$$

Thus $F(x) = 40x$ and $W = \int_0^{0.5} 40x \, dx$.

4. Integrate and evaluate.

$$W = [20x^2]_0^{0.5} = 20(0.5)^2 - 0 = 20(0.25) = 5.$$

5. State and check the work.

$$\boxed{W = 5 \text{ J}}.$$

The force rises linearly from 0 to 20 N, so the triangular area is also $\frac{1}{2}(0.5)(20) = 5 \text{ J}$.