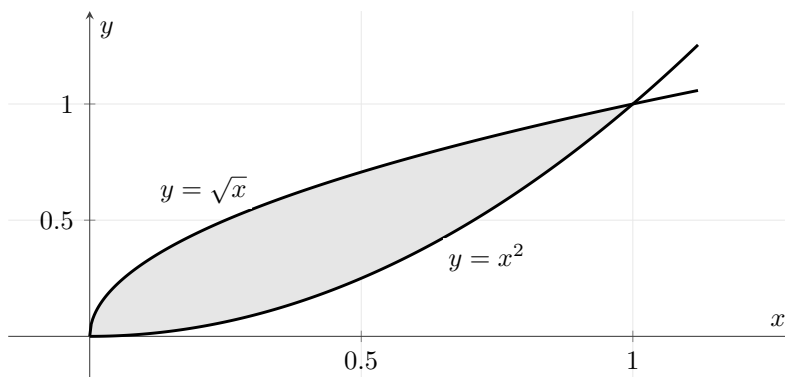


Question 10

The base of a solid is the region bounded by

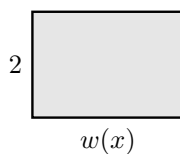
$$y = \sqrt{x} \quad \text{and} \quad y = x^2,$$

between their points of intersection. Cross-sections parallel to the y -axis are rectangles whose width is equal to the distance between the curves and whose height is constant and equal to 2. Find the volume of the solid.



Shaded base region in the xy -plane.

Cross-section perpendicular to the x -axis



Schematic cross-section; dimensions vary with x .

Question 10 – Solution

1. Interpret the slice direction and find the bounds.

The base segments are parallel to the y -axis, so they are vertical; integrate with respect to x . Solving $\sqrt{x} = x^2$ gives $x = x^4$, or $x(x^3 - 1) = 0$. The real intersections occur at $x = 0, 1$.

2. Identify width and height.

For $0 \leq x \leq 1$, $\sqrt{x} \geq x^2$. Thus each rectangle has

$$w(x) = \sqrt{x} - x^2, \quad h = 2.$$

The height of 2 extends out of the base plane.

3. Find the area and set up the integral.

$$A(x) = 2(\sqrt{x} - x^2), \quad V = \int_0^1 A(x) dx = 2 \int_0^1 (x^{1/2} - x^2) dx.$$

4. Integrate using the power rule.

$$\int x^{1/2} dx = \frac{2}{3}x^{3/2}, \quad \int x^2 dx = \frac{x^3}{3}.$$
$$V = 2 \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3} \right]_0^1.$$

5. Apply the bounds and simplify.

$$V = 2 \left[\left(\frac{2}{3} - \frac{1}{3} \right) - 0 \right] = \boxed{\frac{2}{3}}.$$

All volumes are in cubic units.