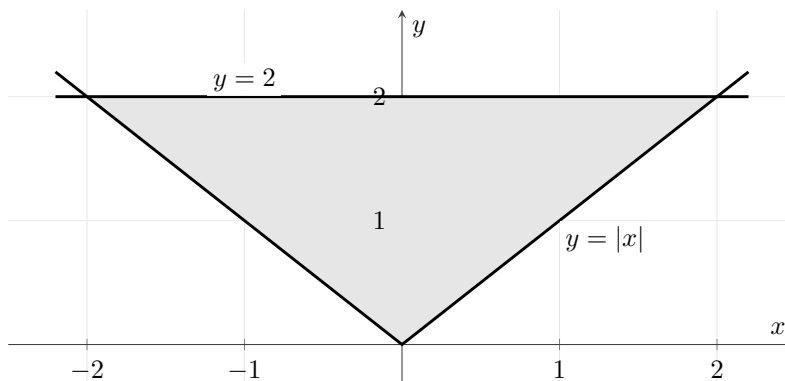


Question 9

The base of a solid is the region bounded by

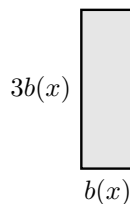
$$y = |x| \quad \text{and} \quad y = 2,$$

between their intersection points. Cross-sections perpendicular to the x -axis are rectangles whose height is three times their base. Find the volume of the solid.



Shaded base region in the xy -plane.

Cross-section perpendicular to the x -axis



Schematic cross-section; dimensions vary with x .

Question 9 – Solution**1. Find the bounds.**

The intersections satisfy $|x| = 2$, giving $x = -2$ and $x = 2$. Use dx slices because the cross-sections are perpendicular to the x -axis.

2. Identify the rectangle dimensions.

The base length is top minus bottom, and the height is three times the base:

$$b(x) = 2 - |x|, \quad h(x) = 3(2 - |x|).$$

3. Set up the area and volume.

$$A(x) = b(x)h(x) = 3(2 - |x|)^2, \quad V = 3 \int_{-2}^2 (2 - |x|)^2 dx.$$

4. Use symmetry and integrate.

The integrand is even; on $[0, 2]$, $|x| = x$. Thus

$$V = 6 \int_0^2 (2 - x)^2 dx = 6 \int_0^2 (4 - 4x + x^2) dx.$$

$$V = 6 \left[4x - 2x^2 + \frac{x^3}{3} \right]_0^2.$$

5. Apply the bounds.

$$V = 6 \left[\left(8 - 8 + \frac{8}{3} \right) - 0 \right] = 6 \cdot \frac{8}{3} = \boxed{16}.$$

All volumes are in cubic units.