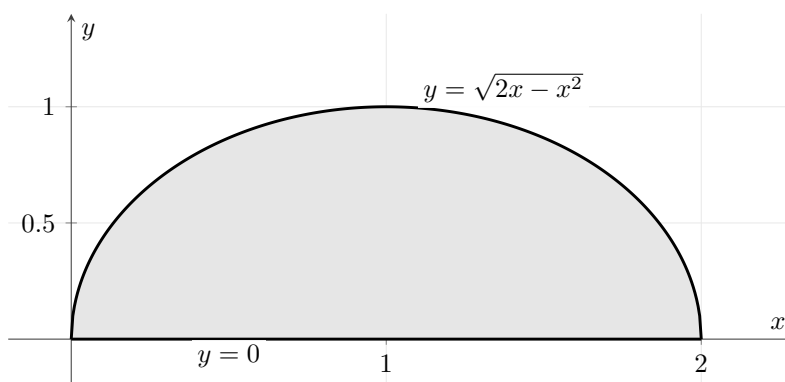


Question 8

The base of a solid is the region bounded by

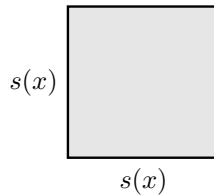
$$y = \sqrt{2x - x^2} \quad \text{and} \quad y = 0.$$

Cross-sections perpendicular to the x -axis are squares. Find the volume of the solid.



Shaded base region in the xy -plane.

Cross-section perpendicular to the x -axis



Schematic cross-section; dimensions vary with x .

Question 8 – Solution**1. Determine the bounds.**

The square root is real when $2x - x^2 = x(2 - x) \geq 0$, giving $0 \leq x \leq 2$. These endpoints are also the intersections with $y = 0$.

2. Find the square side length.

The cross-sections are perpendicular to the x -axis. Their side length is the vertical segment in the base:

$$s(x) = \sqrt{2x - x^2} - 0.$$

3. Find the cross-sectional area.

$$A(x) = s(x)^2 = \left(\sqrt{2x - x^2}\right)^2 = 2x - x^2.$$

4. Set up and integrate the volume.

$$V = \int_0^2 A(x) dx = \int_0^2 (2x - x^2) dx = \left[x^2 - \frac{x^3}{3} \right]_0^2.$$

5. Evaluate and simplify.

$$V = \left(4 - \frac{8}{3} \right) - 0 = \frac{12 - 8}{3} = \boxed{\frac{4}{3}}.$$

All volumes are in cubic units.