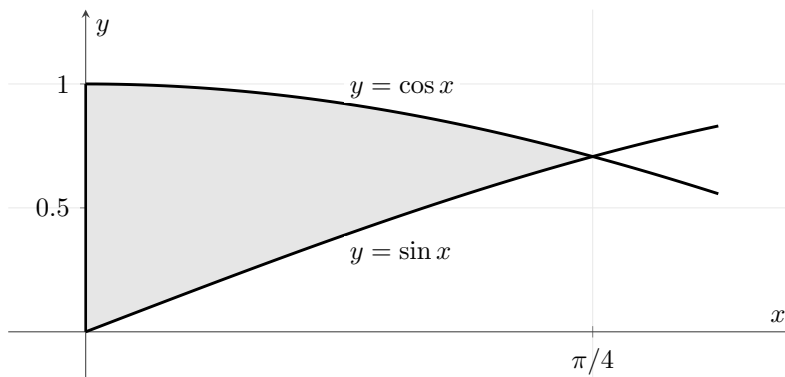


Question 6

The base of a solid is the region bounded by

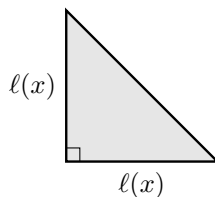
$$y = \cos x \quad \text{and} \quad y = \sin x$$

on the interval where $\cos x \geq \sin x$ in the first quadrant. Cross-sections perpendicular to the x -axis are isosceles right triangles with one leg lying in the base region. Find the volume of the solid.



Shaded base region in the xy -plane.

Cross-section perpendicular to the x -axis



Schematic cross-section; dimensions vary with x .

Question 6 – Solution**1. Determine the bounds.**

In the first quadrant, $\cos x = \sin x$ at $x = \pi/4$. The specified order $\cos x \geq \sin x$ holds for $0 \leq x \leq \pi/4$.

2. Identify the triangle leg and area.

The segment in the base is a leg, not the hypotenuse. Both perpendicular legs have length

$$\ell(x) = \cos x - \sin x, \quad A(x) = \frac{1}{2}\ell(x)^2.$$

3. Set up the volume and simplify.

$$V = \frac{1}{2} \int_0^{\pi/4} (\cos x - \sin x)^2 dx.$$
$$(\cos x - \sin x)^2 = \cos^2 x + \sin^2 x - 2 \sin x \cos x = 1 - \sin(2x).$$

4. Integrate each term.

$$V = \frac{1}{2} \int_0^{\pi/4} (1 - \sin(2x)) dx = \frac{1}{2} \left[x + \frac{1}{2} \cos(2x) \right]_0^{\pi/4}.$$

5. Apply the bounds.

$$V = \frac{1}{2} \left[\left(\frac{\pi}{4} + \frac{1}{2} \cos \frac{\pi}{2} \right) - \left(0 + \frac{1}{2} \cos 0 \right) \right] = \boxed{\frac{\pi}{8} - \frac{1}{4}}.$$

All volumes are in cubic units.