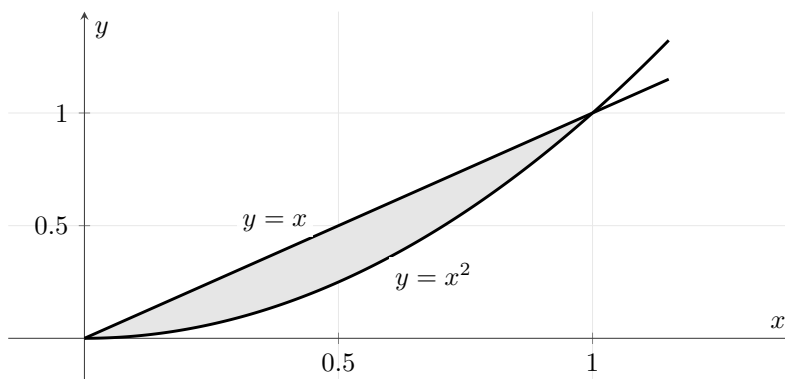


Question 5

The base of a solid is the region bounded by

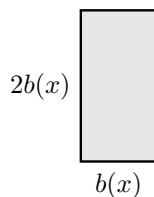
$$y = x \quad \text{and} \quad y = x^2$$

between their intersection points. Cross-sections perpendicular to the x -axis are rectangles whose height is *twice* the base in the region. Find the volume of the solid.



Shaded base region in the xy -plane.

Cross-section perpendicular to the x -axis



Schematic cross-section; dimensions vary with x .

Question 5 – Solution**1. Find the interval and upper curve.**

$$x = x^2 \implies x(x - 1) = 0 \implies x = 0, 1.$$

On $[0, 1]$, $x \geq x^2$, so the line lies above the parabola.

2. Identify the rectangle dimensions.

The base is the vertical distance between curves; its height is twice that distance:

$$b(x) = x - x^2, \quad h(x) = 2b(x) = 2(x - x^2).$$

3. Find the area and expand.

$$A(x) = b(x)h(x) = 2(x - x^2)^2 = 2x^2 - 4x^3 + 2x^4.$$

4. Set up and integrate the volume.

$$V = \int_0^1 A(x) dx = \left[\frac{2}{3}x^3 - x^4 + \frac{2}{5}x^5 \right]_0^1.$$

Each term follows from $\int x^n dx = x^{n+1}/(n+1)$.

5. Evaluate and combine the fractions.

$$V = \left(\frac{2}{3} - 1 + \frac{2}{5} \right) - 0 = \frac{10 - 15 + 6}{15} = \boxed{\frac{1}{15}}.$$

All volumes are in cubic units.