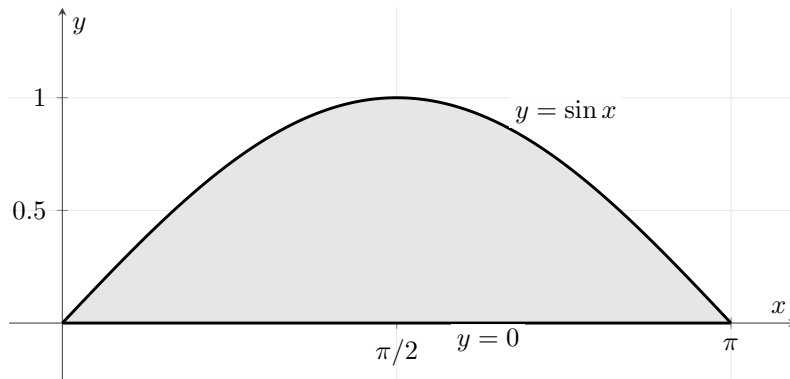


**Question 3**

The base of a solid is the region enclosed by

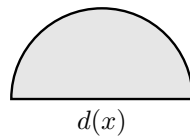
$$y = \sin x \quad \text{and} \quad y = 0$$

from  $x = 0$  to  $x = \pi$ . Cross-sections perpendicular to the  $x$ -axis are semicircles whose diameters lie in the base region. Find the volume of the solid.



Shaded base region in the  $xy$ -plane.

**Cross-section perpendicular to the  $x$ -axis**



Schematic cross-section; dimensions vary with  $x$ .

**Question 3 – Solution****1. Identify the diameter and radius.**

Use  $dx$  slices for  $0 \leq x \leq \pi$ . The base segment is a diameter, so

$$d(x) = \sin x - 0 = \sin x, \quad r(x) = \frac{d(x)}{2} = \frac{\sin x}{2}.$$

**2. Find the area of a semicircle.**

$$A(x) = \frac{1}{2}\pi r(x)^2 = \frac{1}{2}\pi \left(\frac{\sin x}{2}\right)^2 = \frac{\pi}{8} \sin^2 x.$$

**3. Set up the volume and use a trigonometric identity.**

$$V = \frac{\pi}{8} \int_0^\pi \sin^2 x \, dx, \quad \sin^2 x = \frac{1 - \cos(2x)}{2}.$$
$$V = \frac{\pi}{16} \int_0^\pi (1 - \cos(2x)) \, dx.$$

**4. Integrate with the chain-rule factor.**

Since  $\int \cos(2x) \, dx = \sin(2x)/2$ ,

$$V = \frac{\pi}{16} \left[ x - \frac{\sin(2x)}{2} \right]_0^\pi.$$

**5. Apply the bounds.**

$$V = \frac{\pi}{16} \left[ \left( \pi - \frac{\sin(2\pi)}{2} \right) - \left( 0 - \frac{\sin 0}{2} \right) \right] = \boxed{\frac{\pi^2}{16}}.$$

All volumes are in cubic units.