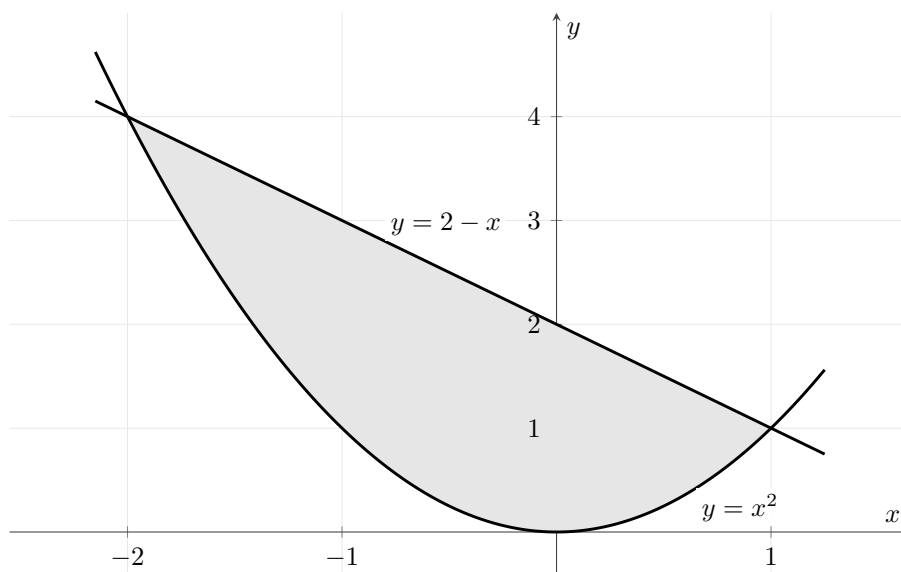


Question 1

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2, \quad y = 2 - x,$$

about the x -axis.



Rotate the shaded region about the x -axis.

Question 1 – Solution**1. Find the intersections.**

$$x^2 = 2 - x \implies x^2 + x - 2 = (x + 2)(x - 1) = 0.$$

Thus $x = -2$ and $x = 1$, giving intersection points $(-2, 4)$ and $(1, 1)$.

2. Choose washers and identify the radii.

Use vertical slices perpendicular to the x -axis. On $[-2, 1]$, $0 \leq x^2 \leq 2 - x$, so

$$R(x) = 2 - x, \quad r(x) = x^2.$$

3. Set up the volume and expand.

$$V = \pi \int_{-2}^1 (R^2 - r^2) dx = \pi \int_{-2}^1 ((2 - x)^2 - x^4) dx.$$
$$(2 - x)^2 - x^4 = 4 - 4x + x^2 - x^4.$$

4. Find an antiderivative.

Using the power rule for each term,

$$F(x) = 4x - 2x^2 + \frac{x^3}{3} - \frac{x^5}{5}, \quad V = \pi[F(x)]_{-2}^1.$$

5. Evaluate the endpoints and subtract.

$$F(1) = 4 - 2 + \frac{1}{3} - \frac{1}{5} = \frac{32}{15}, \quad F(-2) = -8 - 8 - \frac{8}{3} + \frac{32}{5} = -\frac{184}{15}.$$

$$V = \pi \left(\frac{32}{15} + \frac{184}{15} \right) = \boxed{\frac{72\pi}{5}}.$$

All volumes are in cubic units.