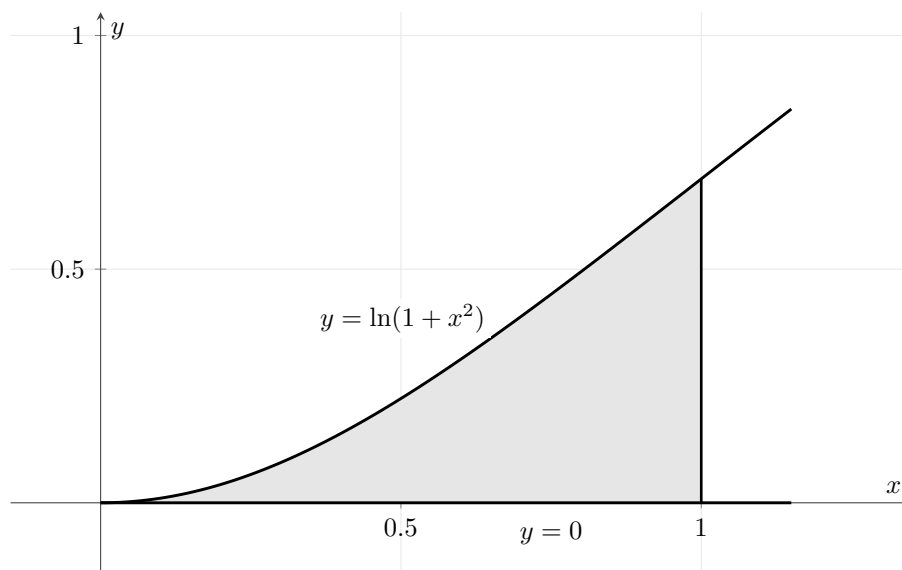


Question 10

Find the volume of the solid obtained by rotating the region bounded by

$$y = \ln(1 + x^2) \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = 1$, about the y -axis.



Rotate the shaded region about the y -axis.

Question 10 – Solution**1. Identify shell dimensions and set up the integral.**

Use vertical shells about the y -axis, with $0 \leq x \leq 1$:

$$r(x) = x, \quad h(x) = \ln(1 + x^2).$$

$$V = 2\pi \int_0^1 x \ln(1 + x^2) dx.$$

2. Substitute and change the bounds.

Let $u = 1 + x^2$. Then $du = 2x dx$, so $x dx = du/2$. The bounds change as follows:

$$x = 0 \Rightarrow u = 1, \quad x = 1 \Rightarrow u = 2.$$

3. Rewrite the volume in terms of u .

$$V = 2\pi \cdot \frac{1}{2} \int_1^2 \ln u du = \pi \int_1^2 \ln u du.$$

4. Integrate the logarithm by parts.

Choose $a = \ln u$ and $db = du$, giving $da = du/u$ and $b = u$:

$$\int \ln u du = u \ln u - \int u \frac{1}{u} du = u \ln u - u.$$

5. Evaluate the endpoints.

$$\begin{aligned} V &= \pi [u \ln u - u]_1^2 \\ &= \pi [(2 \ln 2 - 2) - (\ln 1 - 1)] \\ &= \pi(2 \ln 2 - 1) = \boxed{2\pi \left(\ln 2 - \frac{1}{2} \right)}. \end{aligned}$$

All volumes are in cubic units.