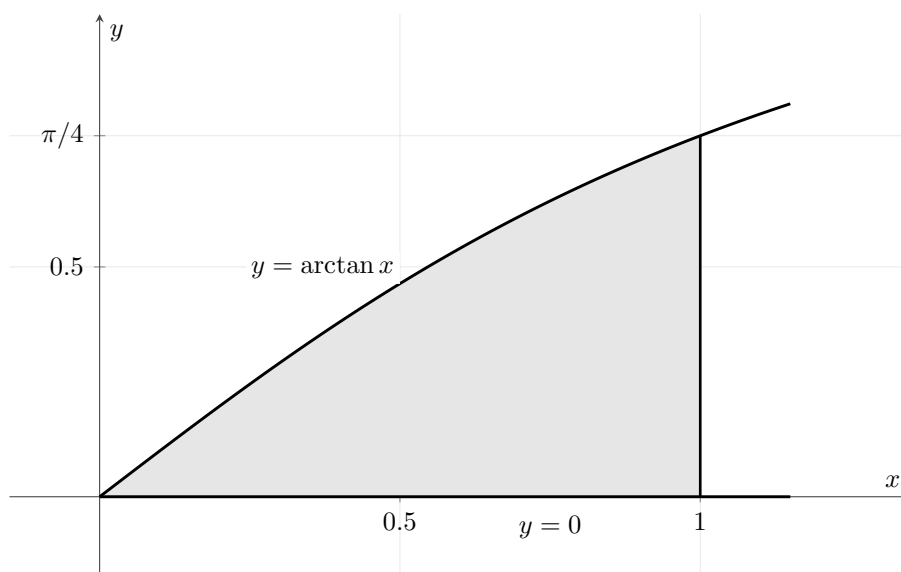


**Question 9**

Find the volume of the solid obtained by rotating the region bounded by

$$y = \arctan x \quad \text{and} \quad y = 0,$$

from  $x = 0$  to  $x = 1$ , about the  $y$ -axis.



Rotate the shaded region about the  $y$ -axis.

**Question 9 – Solution****1. Identify shell radius and height.**

Use vertical slices about the  $y$ -axis on  $[0, 1]$ :

$$r(x) = x, \quad h(x) = \arctan x.$$

Angles are measured in radians.

**2. Set up the volume integral.**

$$V = 2\pi \int_0^1 x \arctan x \, dx.$$

**3. Apply integration by parts.**

Let  $u = \arctan x$ ,  $dv = x \, dx$ , so  $du = dx/(1 + x^2)$  and  $v = x^2/2$ :

$$\int x \arctan x \, dx = \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2}{1 + x^2} \, dx.$$

**4. Simplify the remaining rational function.**

$$\frac{x^2}{1 + x^2} = 1 - \frac{1}{1 + x^2}.$$

Since  $\int (1 + x^2)^{-1} dx = \arctan x$ , an antiderivative is

$$F(x) = \frac{x^2}{2} \arctan x - \frac{x}{2} + \frac{1}{2} \arctan x.$$

**5. Apply the bounds and simplify.**

Using  $\arctan 1 = \pi/4$  and  $\arctan 0 = 0$ ,

$$F(1) - F(0) = \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2}.$$

$$V = 2\pi \left( \frac{\pi}{4} - \frac{1}{2} \right) = \boxed{\pi \left( \frac{\pi}{2} - 1 \right)}.$$

All volumes are in cubic units.