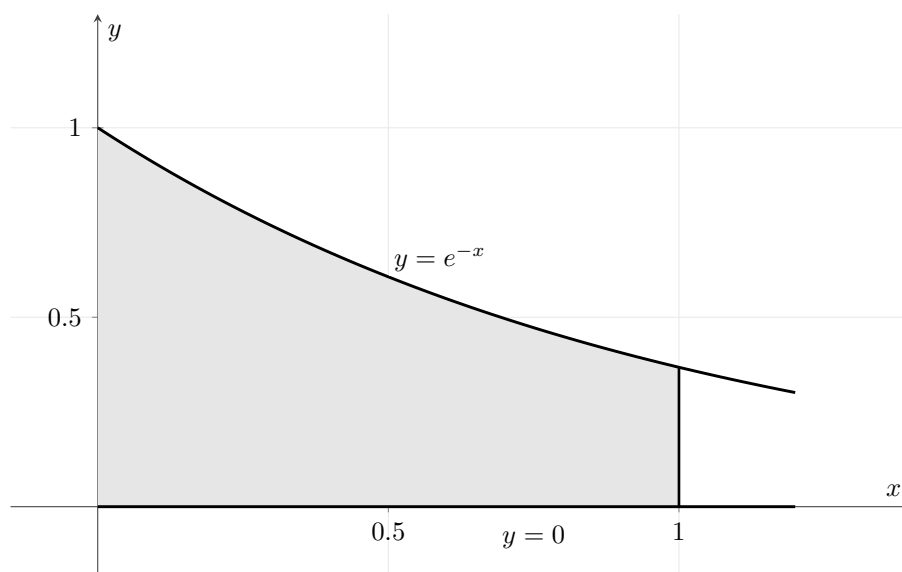


Question 8

Find the volume of the solid obtained by rotating the region bounded by

$$y = e^{-x}, \quad y = 0,$$

from $x = 0$ to $x = 1$, about the y -axis.



Rotate the shaded region about the y -axis.

Question 8 – Solution

1. Identify the shell radius and height.

Use vertical slices over $0 \leq x \leq 1$, parallel to the y -axis:

$$r(x) = x, \quad h(x) = e^{-x} - 0 = e^{-x}.$$

2. Set up the volume integral.

Circumference times height times thickness gives

$$V = 2\pi \int_0^1 r(x)h(x) dx = 2\pi \int_0^1 xe^{-x} dx.$$

3. Apply integration by parts.

Let $u = x$ and $dv = e^{-x}dx$, so $du = dx$ and $v = -e^{-x}$:

$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x}.$$

4. Evaluate the definite integral.

Factor the antiderivative and substitute the bounds:

$$\int_0^1 xe^{-x} dx = [-(x+1)e^{-x}]_0^1 = -\frac{2}{e} - (-1) = 1 - \frac{2}{e}.$$

5. Multiply by the circumference factor.

$$V = \boxed{2\pi \left(1 - \frac{2}{e}\right)}.$$

Since $e > 2$, the expression is positive, as a volume must be.

All volumes are in cubic units.