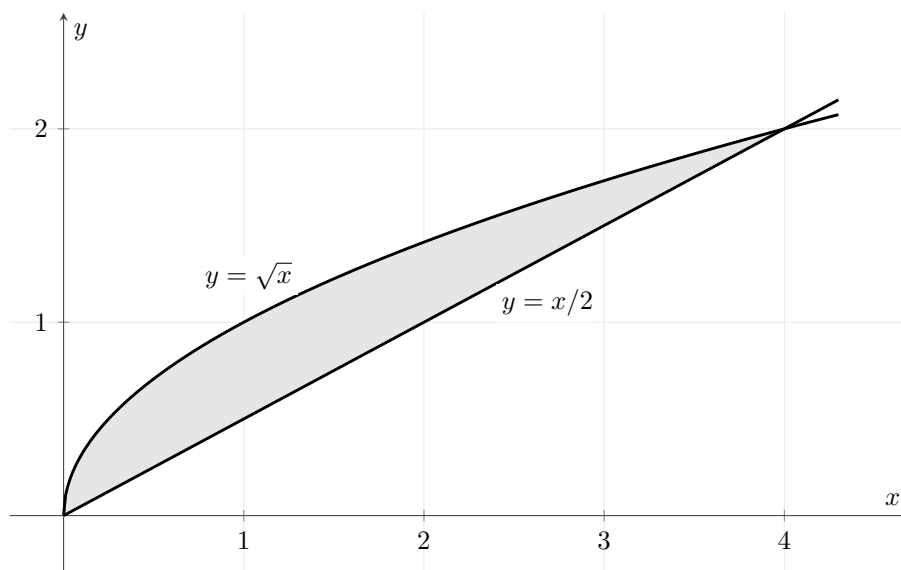


Question 7

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x}, \quad y = \frac{x}{2},$$

between their points of intersection, about the y -axis.



Rotate the shaded region about the y -axis.

Question 7 – Solution

1. Find the intersections.

Both sides of $\sqrt{x} = x/2$ are nonnegative. Squaring and factoring gives

$$x = \frac{x^2}{4} \implies x(x - 4) = 0 \implies x = 0, 4.$$

Both values satisfy the original equation.

2. Identify radius and height.

Use vertical shells about the y -axis. On $[0, 4]$, $\sqrt{x} \geq x/2$:

$$r(x) = x, \quad h(x) = \sqrt{x} - \frac{x}{2}.$$

3. Set up and expand the shell integral.

$$V = 2\pi \int_0^4 x \left(\sqrt{x} - \frac{x}{2} \right) dx = 2\pi \int_0^4 \left(x^{3/2} - \frac{x^2}{2} \right) dx.$$

4. Integrate using the power rule.

$$\int x^{3/2} dx = \frac{2}{5} x^{5/2}, \quad \int \frac{x^2}{2} dx = \frac{x^3}{6}.$$

$$V = 2\pi \left[\frac{2}{5} x^{5/2} - \frac{x^3}{6} \right]_0^4.$$

5. Apply the bounds and simplify.

Since $4^{5/2} = (\sqrt{4})^5 = 32$,

$$V = 2\pi \left(\frac{64}{5} - \frac{64}{6} \right) = 2\pi \left(\frac{192 - 160}{15} \right) = \boxed{\frac{64\pi}{15}}.$$

All volumes are in cubic units.