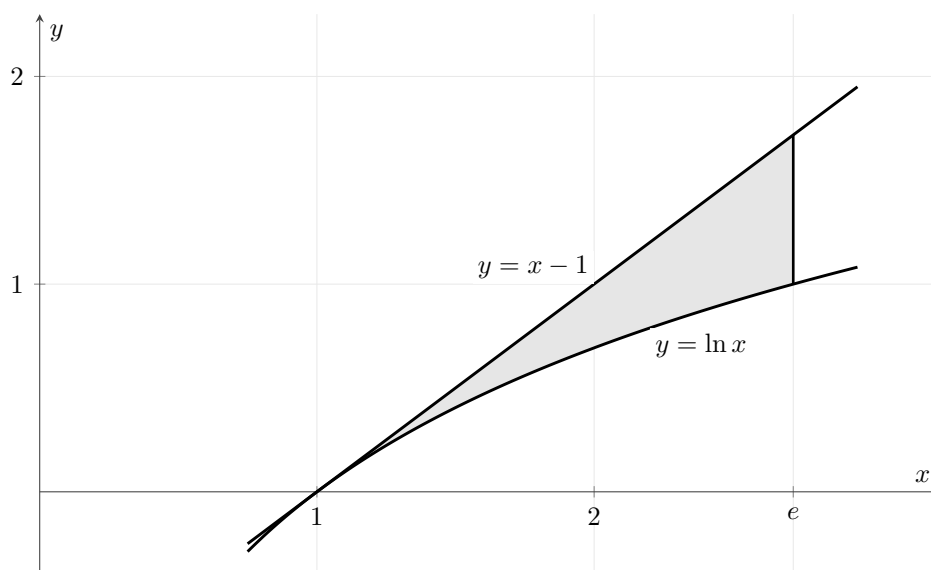


**Question 6**

Use cylindrical shells to find the volume obtained by revolving the region bounded by

$$y = \ln x, \quad y = x - 1, \quad x = 1, \quad x = e$$

about the  $y$ -axis.



Rotate the shaded region about the  $y$ -axis.

## Question 6 – Solution

### 1. Determine the upper curve.

Let  $g(x) = x - 1 - \ln x$ . On  $[1, e]$ ,  $g(1) = 0$  and  $g'(x) = 1 - 1/x \geq 0$ . Hence  $x - 1 \geq \ln x$ .

### 2. Identify shell dimensions and set up the volume.

Use vertical slices parallel to the  $y$ -axis:

$$r(x) = x, \quad h(x) = x - 1 - \ln x.$$

$$V = 2\pi \int_1^e x(x - 1 - \ln x) dx = 2\pi \int_1^e (x^2 - x - x \ln x) dx.$$

### 3. Integrate the logarithmic term by parts.

Let  $u = \ln x$ ,  $dv = x dx$ , so  $du = dx/x$  and  $v = x^2/2$ :

$$\int x \ln x dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx = \frac{x^2}{2} \ln x - \frac{x^2}{4}.$$

### 4. Combine the antiderivatives.

$$F(x) = \frac{x^3}{3} - \frac{x^2}{2} - \left( \frac{x^2}{2} \ln x - \frac{x^2}{4} \right) = \frac{x^3}{3} - \frac{x^2}{4} - \frac{x^2}{2} \ln x.$$

### 5. Apply the bounds.

$$F(e) = \frac{e^3}{3} - \frac{3e^2}{4}, \quad F(1) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}.$$

$$V = 2\pi(F(e) - F(1)) = \boxed{\pi \left( \frac{2e^3}{3} - \frac{3e^2}{2} - \frac{1}{6} \right)}.$$

All volumes are in cubic units.