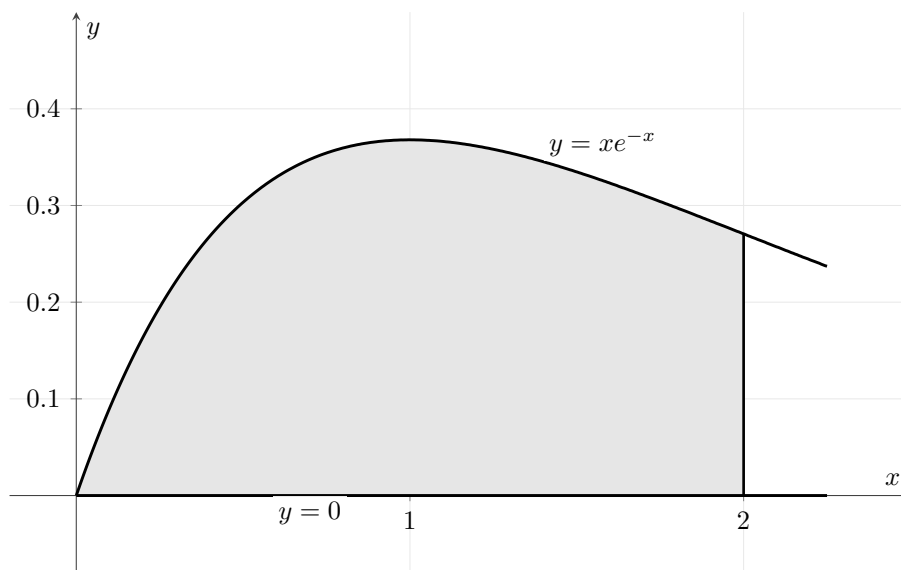


Question 5

Find the volume of the solid obtained by rotating the region bounded by

$$y = xe^{-x} \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = 2$, about the y -axis.



Rotate the shaded region about the y -axis.

Question 5 – Solution

1. Identify the shell dimensions.

Use vertical slices for rotation about the y -axis. On $0 \leq x \leq 2$,

$$r(x) = x, \quad h(x) = xe^{-x}.$$

2. Set up the volume integral.

$$V = 2\pi \int_0^2 r(x)h(x) dx = 2\pi \int_0^2 x^2e^{-x} dx.$$

3. Integrate by parts once.

Let $u = x^2$, $dv = e^{-x}dx$, so $du = 2x dx$ and $v = -e^{-x}$:

$$\int x^2e^{-x} dx = -x^2e^{-x} + 2 \int xe^{-x} dx.$$

4. Integrate by parts a second time.

For the remaining integral, choose $u = x$ and $dv = e^{-x}dx$:

$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x}.$$

Substitution gives $F(x) = -e^{-x}(x^2 + 2x + 2)$.

5. Evaluate at 2 and 0.

$$\begin{aligned} V &= 2\pi \left[-e^{-x}(x^2 + 2x + 2) \right]_0^2 \\ &= 2\pi \left[-10e^{-2} - (-2) \right] \\ &= 2\pi(2 - 10e^{-2}) = \boxed{4\pi(1 - 5e^{-2})}. \end{aligned}$$

All volumes are in cubic units.