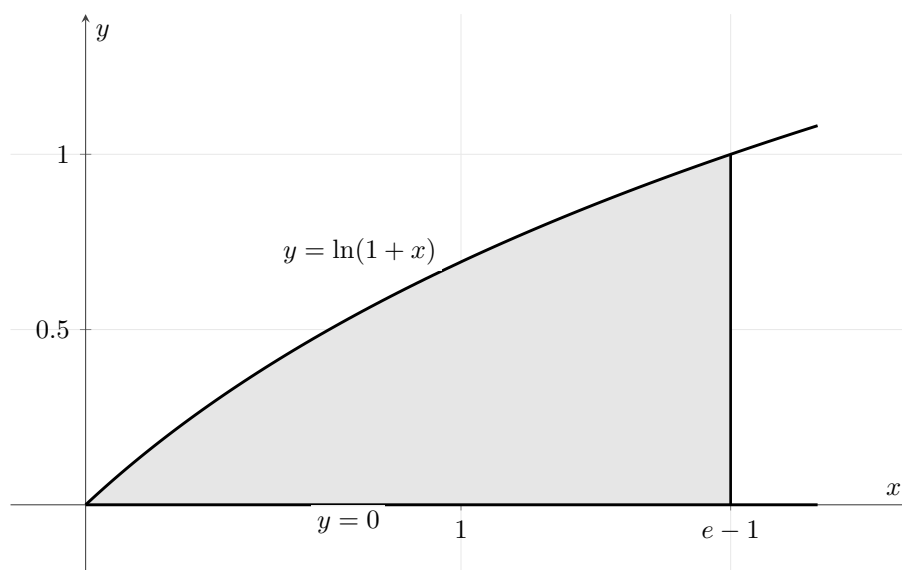


Question 4

Find the volume of the solid obtained by rotating the region bounded by

$$y = \ln(1 + x) \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = e - 1$, about the y -axis.



Rotate the shaded region about the y -axis.

Question 4 – Solution

1. Identify shell radius and height.

Use vertical slices about the y -axis, with $0 \leq x \leq e - 1$:

$$r(x) = x, \quad h(x) = \ln(1 + x) - 0.$$

2. Set up the volume.

$$V = 2\pi \int_0^{e-1} x \ln(1 + x) dx.$$

3. Apply integration by parts.

Choose $u = \ln(1 + x)$ and $dv = x dx$. Then $du = dx/(1 + x)$ and $v = x^2/2$:

$$\int x \ln(1 + x) dx = \frac{x^2}{2} \ln(1 + x) - \frac{1}{2} \int \frac{x^2}{1 + x} dx.$$

4. Divide and integrate the remaining terms.

Since $x^2 = (x + 1)(x - 1) + 1$,

$$\frac{x^2}{1 + x} = x - 1 + \frac{1}{1 + x}.$$

An antiderivative is therefore

$$F(x) = \frac{x^2}{2} \ln(1 + x) - \frac{x^2}{4} + \frac{x}{2} - \frac{1}{2} \ln(1 + x).$$

5. Apply the bounds and multiply by 2π .

Using $\ln e = 1$ and $F(0) = 0$,

$$\begin{aligned} F(e - 1) - F(0) &= \frac{(e - 1)^2}{4} + \frac{e - 1}{2} - \frac{1}{2} = \frac{e^2 - 3}{4}, \\ V &= 2\pi \cdot \frac{e^2 - 3}{4} = \boxed{\frac{\pi}{2}(e^2 - 3)}. \end{aligned}$$

All volumes are in cubic units.