

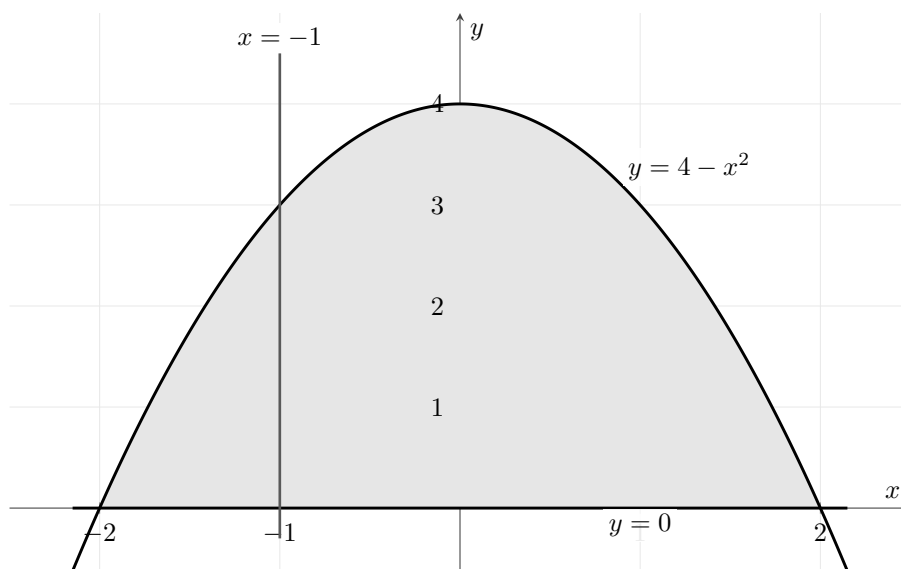
Question 3

Find the volume of the solid obtained by rotating the region bounded by

$$y = 4 - x^2 \quad \text{and} \quad y = 0,$$

about the vertical line

$$x = -1.$$



Rotate the shaded region about the line $x = -1$.

Question 3 – Solution**1. Locate the region and rotation axis.**

The parabola meets $y = 0$ at $x = \pm 2$. The axis $x = -1$ passes through the region. Vertical strips on opposite sides can rotate into the same shell.

2. Count each radius once.

Let r be distance from $x = -1$. The right strip is at $x = -1 + r$, with $0 \leq r \leq 3$. Its height is $4 - (r - 1)^2$. For $0 \leq r \leq 1$, a left strip also exists at $x = -1 - r$, with height $4 - (r + 1)^2$.

$$[4 - (r - 1)^2] - [4 - (r + 1)^2] = 4r \geq 0.$$

Both strips start at $y = 0$, so the taller right strip includes the entire left strip after rotation. Use only the right strip.

3. Set up the shell integral.

$$\begin{aligned} h(r) &= 4 - (r - 1)^2 = 3 + 2r - r^2. \\ V &= 2\pi \int_0^3 r(3 + 2r - r^2) dr = 2\pi \int_0^3 (3r + 2r^2 - r^3) dr. \end{aligned}$$

4. Integrate each power.

$$V = 2\pi \left[\frac{3}{2}r^2 + \frac{2}{3}r^3 - \frac{1}{4}r^4 \right]_0^3.$$

5. Apply the bounds.

$$V = 2\pi \left(\frac{27}{2} + 18 - \frac{81}{4} \right) = 2\pi \left(\frac{54 + 72 - 81}{4} \right) = \boxed{\frac{45\pi}{2}}.$$

All volumes are in cubic units.