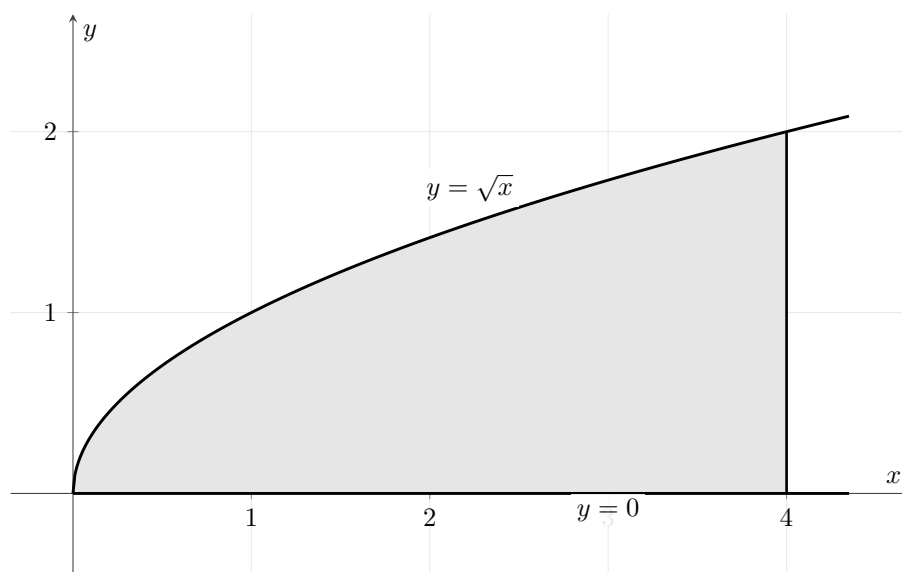


Question 2

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = 4$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 2 – Solution

1. Choose horizontal slices and y -bounds.

Shell slices run parallel to the x -axis. Rewrite $y = \sqrt{x}$ as $x = y^2$. The region extends from $y = 0$ to $y = \sqrt{4} = 2$.

2. Identify the shell radius and height.

The radius is the distance to $y = 0$. For a horizontal slice, shell height is its horizontal length, right minus left:

$$r(y) = y, \quad h(y) = 4 - y^2.$$

3. Set up the shell integral.

A thin shell has volume approximately $(2\pi r)(h) dy$. Thus

$$V = 2\pi \int_0^2 y(4 - y^2) dy = 2\pi \int_0^2 (4y - y^3) dy.$$

4. Integrate with respect to y .

$$\int 4y dy = 2y^2, \quad \int y^3 dy = \frac{y^4}{4}.$$
$$V = 2\pi \left[2y^2 - \frac{y^4}{4} \right]_0^2.$$

5. Evaluate both endpoints.

$$V = 2\pi \left[\left(2(2)^2 - \frac{2^4}{4} \right) - 0 \right] = 2\pi(8 - 4) = \boxed{8\pi}.$$

All volumes are in cubic units.