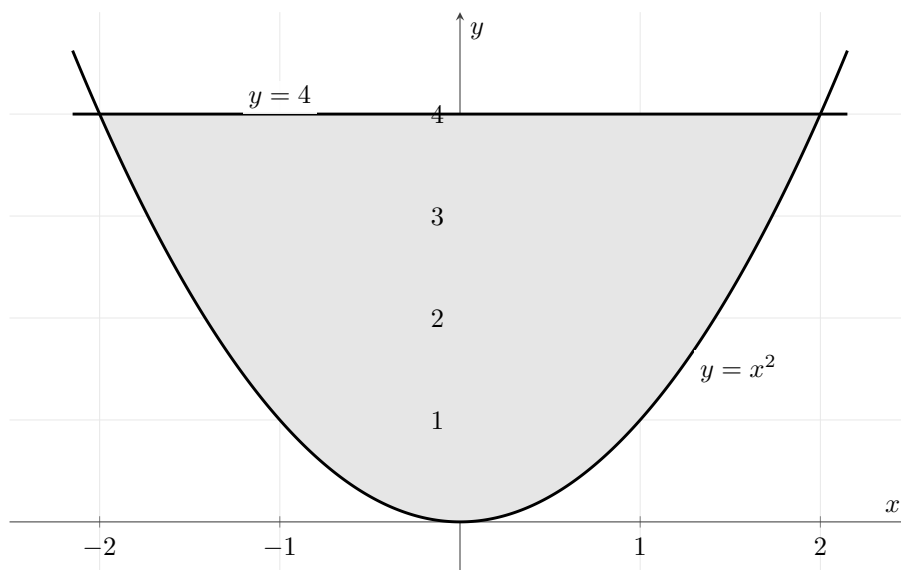


Question 1

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 4$$

about the y -axis.



Rotate the shaded region about the y -axis.

Question 1 – Solution

1. Find the intersections and choose slices.

For shells about the y -axis, use vertical slices parallel to the axis. The curves meet where $x^2 = 4$, so $x = \pm 2$.

2. Choose an interval that counts each shell once.

The half-region $0 \leq x \leq 2$ generates the entire solid. The negative half generates the same shells, so do not double the volume or integrate over both halves.

3. Identify radius, height, and the shell integral.

The radius is the distance to the y -axis; the height is top minus bottom:

$$r(x) = x, \quad h(x) = 4 - x^2.$$
$$V = 2\pi \int_0^2 r(x)h(x) dx = 2\pi \int_0^2 x(4 - x^2) dx.$$

4. Expand and integrate.

$$x(4 - x^2) = 4x - x^3, \quad \int (4x - x^3) dx = 2x^2 - \frac{x^4}{4}.$$

5. Apply the bounds and simplify.

$$\begin{aligned} V &= 2\pi \left[2x^2 - \frac{x^4}{4} \right]_0^2 \\ &= 2\pi \left[\left(8 - \frac{16}{4} \right) - 0 \right] = 2\pi(4) = \boxed{8\pi}. \end{aligned}$$

All volumes are in cubic units.