

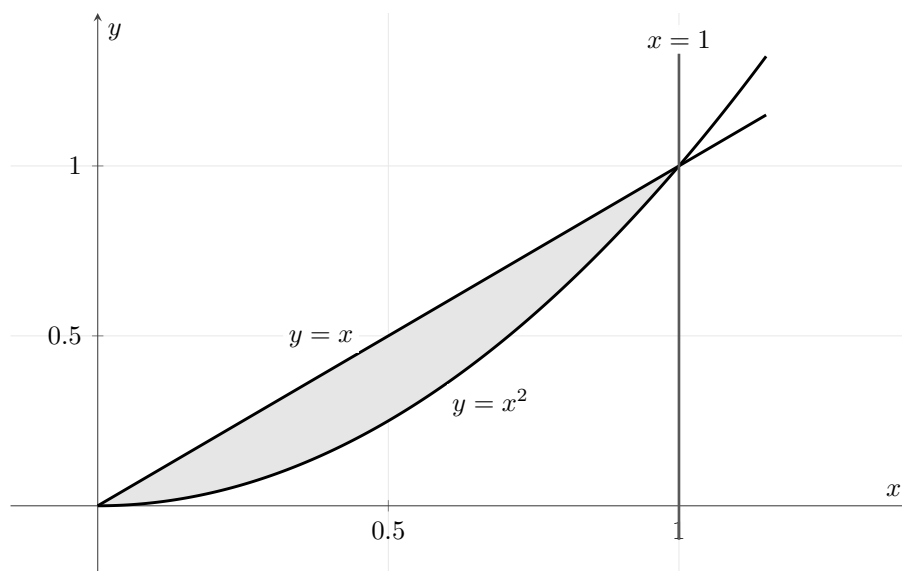
Question 10

Find the volume of the solid obtained by rotating the region bounded by

$$y = x \quad \text{and} \quad y = x^2,$$

about the vertical line

$$x = 1.$$



Rotate the shaded region about the line $x = 1$.

Question 10 – Solution

1. Find the bounds and choose horizontal slices.

The intersections satisfy $x = x^2$, so $x = 0, 1$ and $y = 0, 1$. Rotation about the vertical line $x = 1$ requires horizontal slices of thickness dy .

2. Rewrite the curves and measure radii.

The boundaries are $x = y$ on the left and $x = \sqrt{y}$ on the right. The axis is to their right, so the farther, left boundary gives the outer radius:

$$R(y) = 1 - y, \quad r(y) = 1 - \sqrt{y}.$$

3. Set up the washer integral and expand.

$$\begin{aligned} V &= \pi \int_0^1 [(1 - y)^2 - (1 - \sqrt{y})^2] dy. \\ (1 - y)^2 - (1 - \sqrt{y})^2 &= (1 - 2y + y^2) - (1 - 2\sqrt{y} + y) \\ &= y^2 - 3y + 2y^{1/2}. \end{aligned}$$

4. Integrate using the power rule.

In particular, $\int 2y^{1/2} dy = 2 \cdot \frac{2}{3}y^{3/2} = \frac{4}{3}y^{3/2}$. Therefore,

$$V = \pi \left[\frac{y^3}{3} - \frac{3y^2}{2} + \frac{4}{3}y^{3/2} \right]_0^1.$$

5. Apply the bounds and simplify.

$$V = \pi \left(\frac{1}{3} - \frac{3}{2} + \frac{4}{3} - 0 \right) = \pi \left(\frac{2 - 9 + 8}{6} \right) = \boxed{\frac{\pi}{6}}.$$

All volumes are in cubic units.