

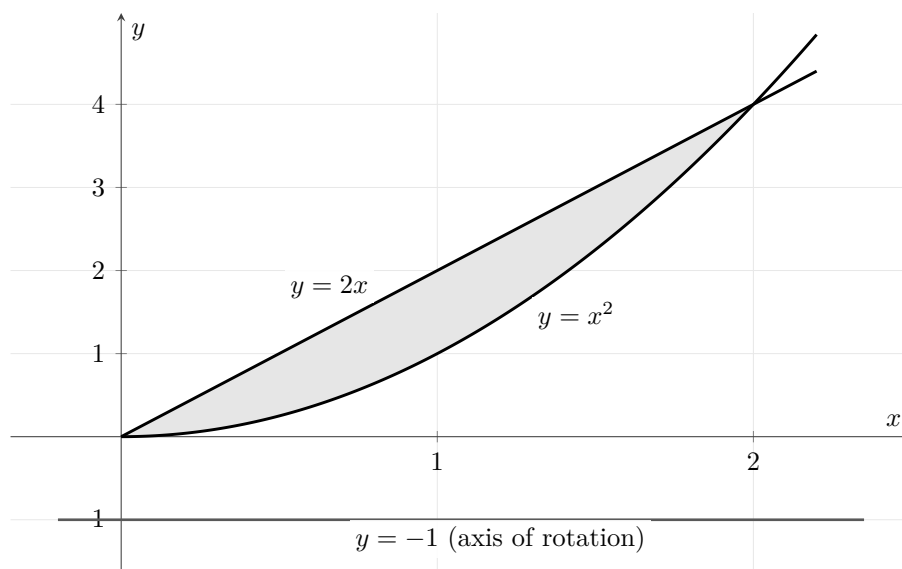
Question 9

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 2x,$$

about the horizontal line

$$y = -1.$$



Rotate the shaded region about the line $y = -1$.

Question 9 – Solution**1. Find the bounds and choose vertical slices.**

The curves meet where $x^2 = 2x$, or $x(x - 2) = 0$. Thus $0 \leq x \leq 2$. The horizontal rotation axis $y = -1$ requires vertical slices of thickness dx .

2. Measure both radii from $y = -1$.

On $[0, 2]$, $2x \geq x^2$. The outer radius reaches the line, and the inner radius reaches the parabola:

$$R(x) = 2x - (-1) = 2x + 1, \quad r(x) = x^2 - (-1) = x^2 + 1.$$

3. Set up the washer integral and expand.

$$\begin{aligned} V &= \pi \int_0^2 [(2x + 1)^2 - (x^2 + 1)^2] dx. \\ (2x + 1)^2 - (x^2 + 1)^2 &= (4x^2 + 4x + 1) - (x^4 + 2x^2 + 1) \\ &= -x^4 + 2x^2 + 4x. \end{aligned}$$

4. Integrate each power.

$$V = \pi \left[-\frac{x^5}{5} + \frac{2x^3}{3} + 2x^2 \right]_0^2.$$

The coefficients follow from $\int x^n dx = x^{n+1}/(n+1)$.

5. Evaluate and use a common denominator.

$$\begin{aligned} V &= \pi \left(-\frac{32}{5} + \frac{16}{3} + 8 - 0 \right) \\ &= \pi \left(\frac{-96 + 80 + 120}{15} \right) = \boxed{\frac{104\pi}{15}}. \end{aligned}$$

All volumes are in cubic units.