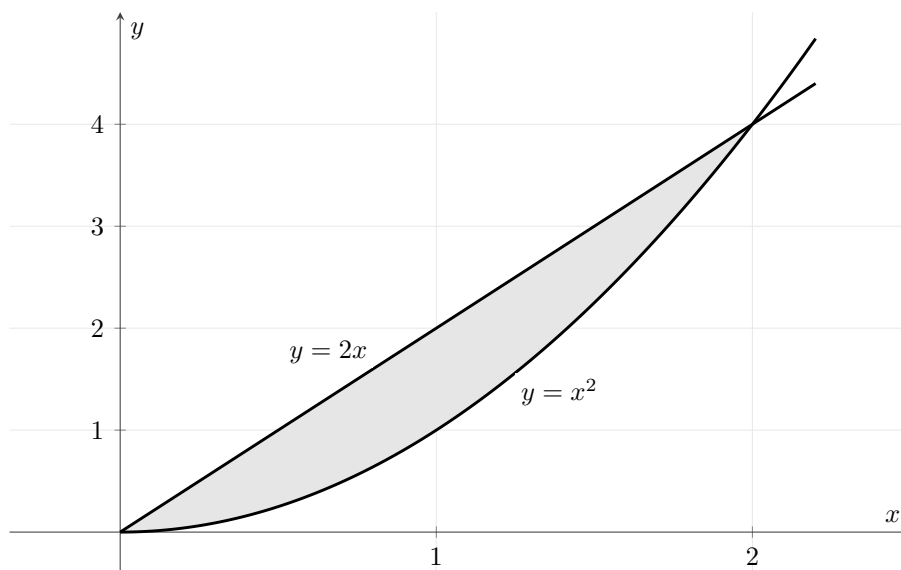


## Question 8

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 2x,$$

about the  $y$ -axis.



Rotate the shaded region about the  $y$ -axis.

**Question 8 – Solution**

**1. Find the intersections and  $y$ -bounds.**

$$x^2 = 2x \implies x(x - 2) = 0 \implies x = 0, 2.$$

The intersection points are  $(0, 0)$  and  $(2, 4)$ , so integrate from  $y = 0$  to  $y = 4$ .

**2. Use horizontal slices and identify radii.**

Rewrite the curves as  $x = \sqrt{y}$  and  $x = y/2$ . For  $0 \leq y \leq 4$ ,  $\sqrt{y} \geq y/2$ . Distances from the  $y$ -axis give

$$R(y) = \sqrt{y}, \quad r(y) = \frac{y}{2}.$$

**3. Set up the washer integral.**

$$V = \pi \int_0^4 \left[ (\sqrt{y})^2 - \left(\frac{y}{2}\right)^2 \right] dy = \pi \int_0^4 \left( y - \frac{y^2}{4} \right) dy.$$

**4. Integrate term by term.**

$$\int y \, dy = \frac{y^2}{2}, \quad \int \frac{y^2}{4} \, dy = \frac{1}{4} \frac{y^3}{3} = \frac{y^3}{12}.$$

$$V = \pi \left[ \frac{y^2}{2} - \frac{y^3}{12} \right]_0^4.$$

**5. Apply the bounds and simplify.**

$$V = \pi \left( \frac{16}{2} - \frac{64}{12} - 0 \right) = \pi \left( 8 - \frac{16}{3} \right) = \boxed{\frac{8\pi}{3}}.$$

All volumes are in cubic units.