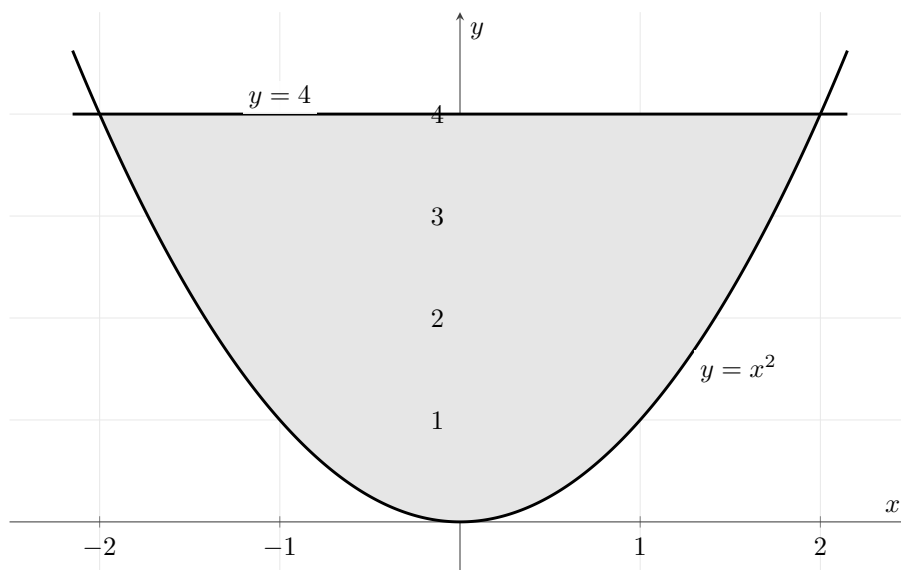


Question 7

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 4,$$

about the y -axis.



Rotate the shaded region about the y -axis.

Question 7 – Solution**1. Use horizontal slices and find the y -bounds.**

Rotation about the y -axis requires slices of thickness dy . The region lies above $y = x^2$ and below $y = 4$, so $0 \leq y \leq 4$.

2. Express the horizontal boundaries in terms of y .

$$y = x^2 \implies x = \pm\sqrt{y}.$$

At height y , the region extends from $-\sqrt{y}$ to $\sqrt{y}$ and crosses the rotation axis.

3. Identify the disk radius.

Rotating the entire horizontal segment gives one disk, not two separate disks. Its radius is the distance from the axis to either edge:

$$R(y) = \sqrt{y}, \quad r(y) = 0.$$

4. Set up the volume and integrate.

$$\begin{aligned} V &= \pi \int_0^4 (R^2 - r^2) dy = \pi \int_0^4 ((\sqrt{y})^2 - 0^2) dy = \pi \int_0^4 y dy. \\ \int y dy &= \frac{y^2}{2}. \end{aligned}$$

5. Apply the bounds.

$$V = \pi \left[\frac{y^2}{2} \right]_0^4 = \pi \left(\frac{16}{2} - 0 \right) = \boxed{8\pi}.$$

All volumes are in cubic units.