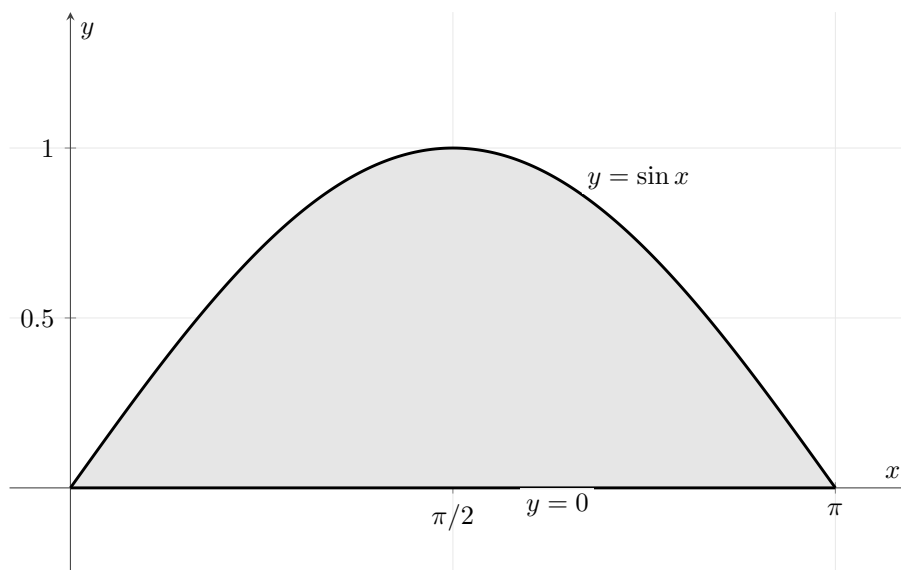


Question 6

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sin x \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = \pi$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 6 – Solution

1. Choose slices and identify the bounds.

Use vertical slices over $0 \leq x \leq \pi$. Since $\sin x \geq 0$ on this interval and the region touches the x -axis, rotation gives disks.

2. Identify radii and set up the volume.

$$R(x) = \sin x, \quad r(x) = 0.$$
$$V = \pi \int_0^\pi ((\sin x)^2 - 0^2) dx = \pi \int_0^\pi \sin^2 x \, dx.$$

3. Use the power-reduction identity.

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \implies V = \frac{\pi}{2} \int_0^\pi (1 - \cos(2x)) \, dx.$$

4. Integrate, including the chain-rule factor.

Because $\frac{d}{dx} \sin(2x) = 2 \cos(2x)$,

$$\int \cos(2x) \, dx = \frac{\sin(2x)}{2}, \quad V = \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^\pi.$$

5. Evaluate at π and 0.

$$V = \frac{\pi}{2} \left[\left(\pi - \frac{\sin(2\pi)}{2} \right) - \left(0 - \frac{\sin 0}{2} \right) \right]$$
$$= \frac{\pi}{2} (\pi - 0) = \boxed{\frac{\pi^2}{2}}.$$

All volumes are in cubic units.