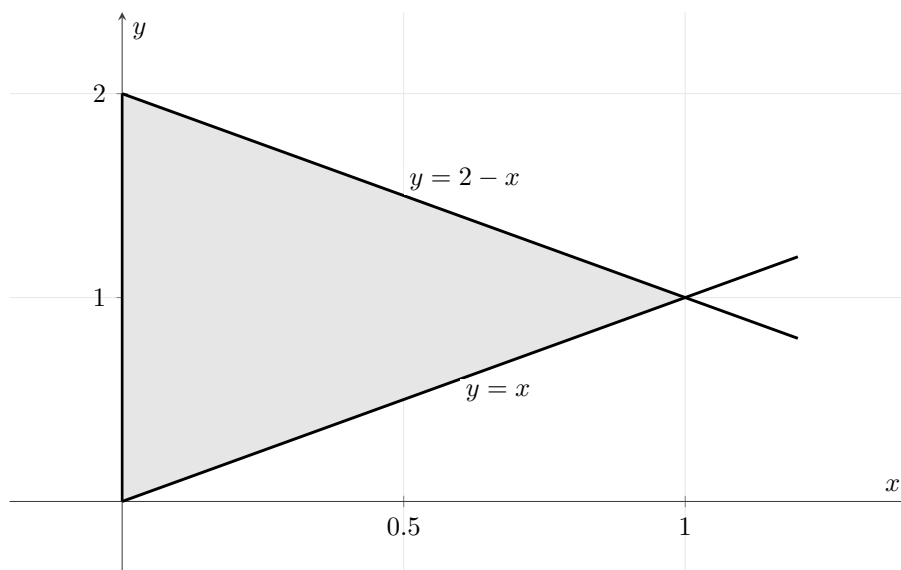


Question 5

Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - x \quad \text{and} \quad y = x,$$

from $x = 0$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 5 – Solution**1. Identify the bounds and slice direction.**

The given left boundary is $x = 0$. The lines meet where $2 - x = x$, giving $x = 1$. Use vertical slices, perpendicular to the x -axis.

2. Measure the radii from the x -axis.

On $[0, 1]$, $2 - x \geq x \geq 0$, so

$$R(x) = 2 - x, \quad r(x) = x.$$

The rotated slice is a washer with area $\pi(R^2 - r^2)$.

3. Set up and simplify the volume integral.

$$\begin{aligned} V &= \pi \int_0^1 [(2 - x)^2 - x^2] dx. \\ (2 - x)^2 - x^2 &= (4 - 4x + x^2) - x^2 = 4 - 4x. \end{aligned}$$

4. Integrate term by term.

$$\begin{aligned} \int 4 dx &= 4x, & \int 4x dx &= 4 \frac{x^2}{2} = 2x^2. \\ V &= \pi [4x - 2x^2]_0^1. \end{aligned}$$

5. Apply the bounds.

$$V = \pi [(4(1) - 2(1)^2) - (4(0) - 2(0)^2)] = \pi(4 - 2) = \boxed{2\pi}.$$

All volumes are in cubic units.