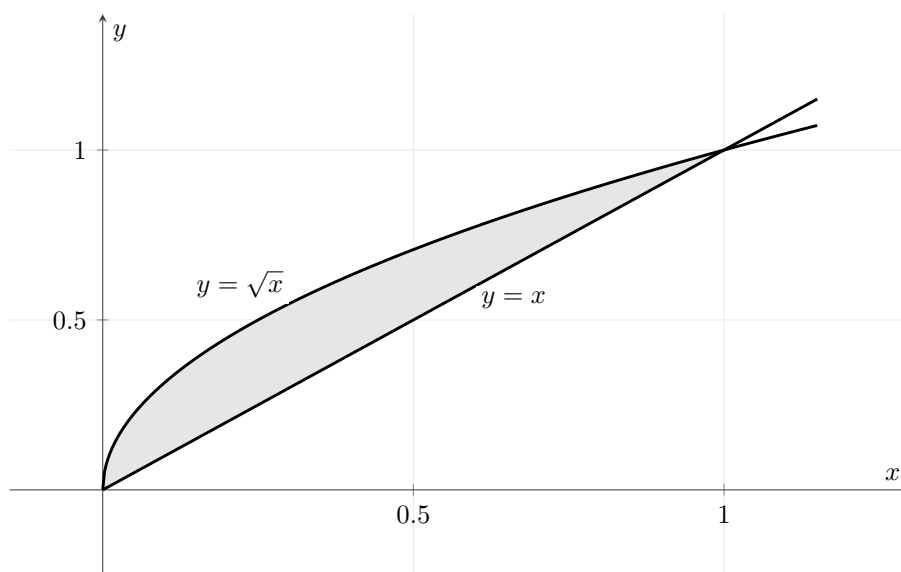


Question 4

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} \quad \text{and} \quad y = x,$$

from $x = 0$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 4 – Solution

1. Find the bounds and choose slices.

The curves meet where $\sqrt{x} = x$. With $x \geq 0$, squaring gives $x = x^2$, so $x = 0$ or $x = 1$. Use vertical slices for rotation about the x -axis.

2. Measure the radii.

For $0 \leq x \leq 1$, $\sqrt{x} \geq x \geq 0$. Thus the square-root curve is farther from the axis:

$$R(x) = \sqrt{x}, \quad r(x) = x.$$

3. Set up the washer integral.

Subtract the squared radii, then multiply by π :

$$V = \pi \int_0^1 (R^2 - r^2) dx = \pi \int_0^1 ((\sqrt{x})^2 - x^2) dx = \pi \int_0^1 (x - x^2) dx.$$

4. Find the antiderivative.

$$\int x dx = \frac{x^2}{2}, \quad \int x^2 dx = \frac{x^3}{3}.$$

$$V = \pi \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1.$$

5. Apply the bounds and simplify.

$$V = \pi \left[\left(\frac{1}{2} - \frac{1}{3} \right) - 0 \right] = \pi \left(\frac{3-2}{6} \right) = \boxed{\frac{\pi}{6}}.$$

All volumes are in cubic units.