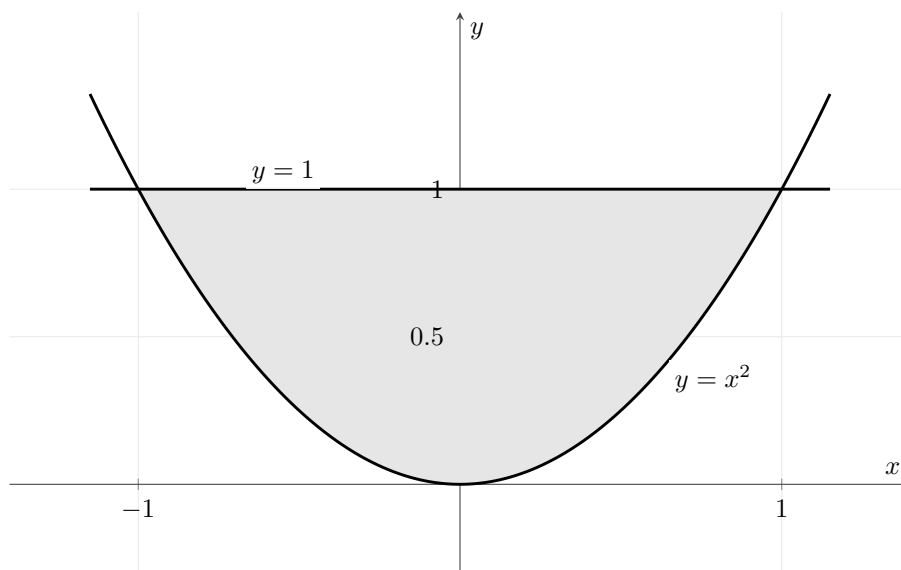


Question 3

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 1,$$

from $x = -1$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 3 – Solution

1. Find the bounds and choose slices.

The intersections satisfy $x^2 = 1$, so $x = -1$ and $x = 1$. Use vertical slices of thickness dx because the rotation axis is horizontal.

2. Identify outer and inner radii.

On $[-1, 1]$, $0 \leq x^2 \leq 1$. Measured from the x -axis,

$$R(x) = 1, \quad r(x) = x^2.$$

The missing inner disk must be subtracted from the outer disk.

3. Set up the washer integral.

$$A(x) = \pi(R(x)^2 - r(x)^2) = \pi(1^2 - (x^2)^2) = \pi(1 - x^4).$$

$$V = \pi \int_{-1}^1 (1 - x^4) dx.$$

4. Integrate term by term.

$$\int 1 dx = x, \quad \int x^4 dx = \frac{x^5}{5}.$$

$$V = \pi \left[x - \frac{x^5}{5} \right]_{-1}^1.$$

5. Evaluate both endpoints.

$$\begin{aligned} V &= \pi \left[\left(1 - \frac{1}{5} \right) - \left(-1 + \frac{1}{5} \right) \right] \\ &= \pi \left(\frac{4}{5} + \frac{4}{5} \right) = \boxed{\frac{8\pi}{5}}. \end{aligned}$$

All volumes are in cubic units.