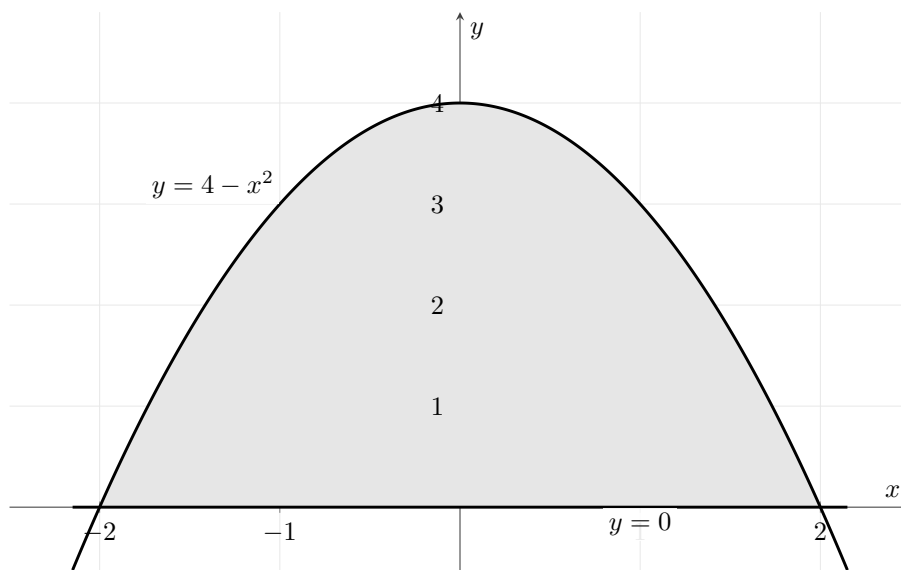


Question 2

Find the volume of the solid obtained by rotating the region bounded by

$$y = 4 - x^2 \quad \text{and} \quad y = 0,$$

from $x = -2$ to $x = 2$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 2 – Solution**1. Choose slices and confirm the bounds.**

Use vertical slices, perpendicular to the x -axis. The curve meets $y = 0$ where

$$4 - x^2 = 0 \implies x = \pm 2.$$

2. Identify the radii.

On $[-2, 2]$, $4 - x^2 \geq 0$. The region touches the axis, so each cross-section is a disk:

$$R(x) = 4 - x^2, \quad r(x) = 0.$$

3. Set up the volume and expand the square.

$$V = \pi \int_{-2}^2 (R^2 - r^2) dx = \pi \int_{-2}^2 (4 - x^2)^2 dx.$$
$$(4 - x^2)^2 = 16 - 8x^2 + x^4.$$

4. Use symmetry and integrate.

The integrand is even, so double the integral over $[0, 2]$:

$$V = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2.$$

5. Substitute and combine the fractions.

$$V = 2\pi \left(32 - \frac{64}{3} + \frac{32}{5} \right)$$
$$= 2\pi \left(\frac{480 - 320 + 96}{15} \right) = \boxed{\frac{512\pi}{15}}.$$

All volumes are in cubic units.