

Question 1

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = 4$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 1 – Solution**1. Choose slices and bounds.**

The rotation axis is horizontal, so use vertical slices of thickness dx . The stated interval is $0 \leq x \leq 4$. Each slice reaches the axis, producing a disk.

2. Measure the radii from the axis.

The distance from $y = 0$ to $y = \sqrt{x}$ is the outer radius. There is no central hole.

$$R(x) = \sqrt{x} - 0 = \sqrt{x}, \quad r(x) = 0.$$

3. Write the cross-sectional area and volume.

$$A(x) = \pi R(x)^2 - \pi r(x)^2 = \pi((\sqrt{x})^2 - 0^2) = \pi x.$$

$$V = \int_0^4 A(x) dx = \pi \int_0^4 x dx.$$

4. Integrate using the power rule.

$$\int x dx = \frac{x^{1+1}}{1+1} = \frac{x^2}{2}, \quad V = \pi \left[\frac{x^2}{2} \right]_0^4.$$

5. Apply the bounds and simplify.

$$V = \pi \left(\frac{4^2}{2} - \frac{0^2}{2} \right) = \pi \left(\frac{16}{2} \right) = \boxed{8\pi}.$$

All volumes are in cubic units.