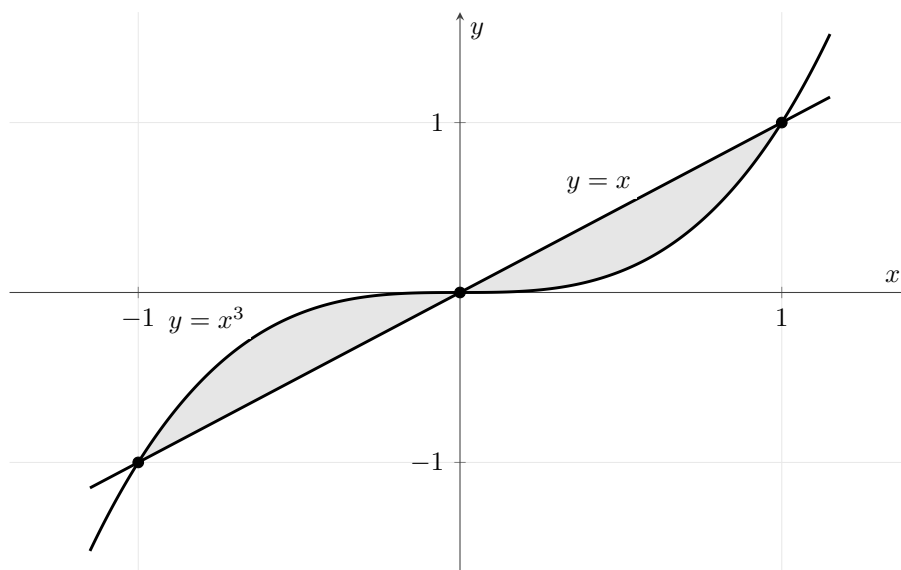


Question 10

Find the exact total area of the regions enclosed by the curves

$$y = x^3 \quad \text{and} \quad y = x,$$

between their points of intersection.



The shaded region is the area to be found.

Question 10 – Solution

1. Find all intersections.

$$x^3 = x \implies x(x-1)(x+1) = 0 \implies x = -1, 0, 1.$$

2. Split the area where the upper curve changes.

On $(-1, 0)$, $x^3 > x$; on $(0, 1)$, $x > x^3$. For example, at $x = -1/2$, $x^3 = -1/8 > -1/2$; at $x = 1/2$, $x = 1/2 > 1/8 = x^3$. Both enclosed regions contribute positive area:

3. Set up upper minus lower on each interval.

$$A = \int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx.$$

4. Integrate and evaluate each region.

The power rule gives $\int x^3 dx = x^4/4$ and $\int x dx = x^2/2$.

$$A_1 = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 = 0 - \left(\frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4},$$

$$A_2 = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

5. Add both areas.

$$A = A_1 + A_2 = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}.$$