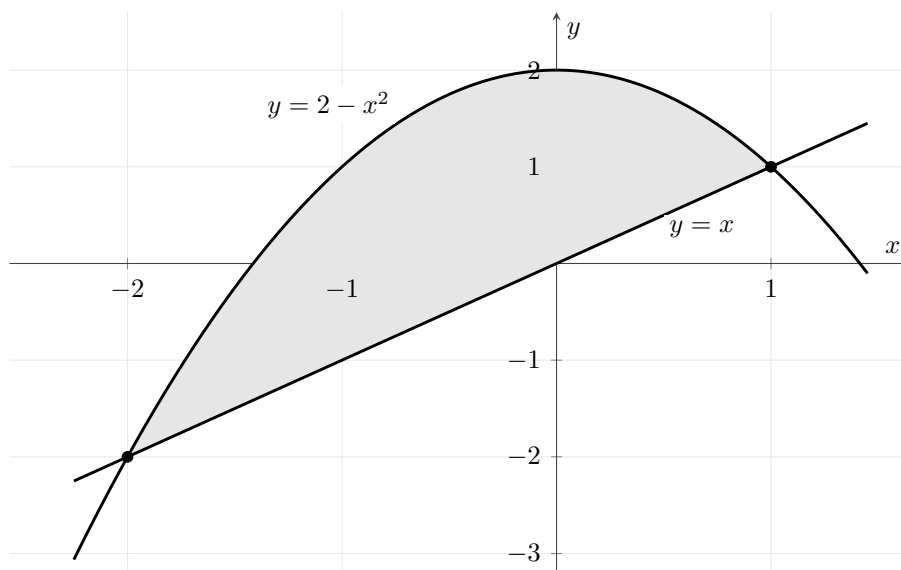


Question 8

Find the exact area of the region enclosed by the curves

$$y = 2 - x^2 \quad \text{and} \quad y = x,$$

between their points of intersection.



The shaded region is the area to be found.

Question 8 – Solution

1. Find the intersections.

$$2 - x^2 = x \implies x^2 + x - 2 = 0 \implies (x + 2)(x - 1) = 0.$$

The zero-product property gives $x + 2 = 0$ or $x - 1 = 0$. Thus $x = -2$ and $x = 1$, giving points $(-2, -2)$ and $(1, 1)$.

2. Determine which curve is above the other.

On $[-2, 1]$, $(2 - x^2) - x = (1 - x)(x + 2) \geq 0$. The parabola is above the line, even where both curves are below the x -axis.

3. Set up upper minus lower.

$$A = \int_{-2}^1 (2 - x - x^2) dx.$$

4. Find an antiderivative.

$$\int 2 dx = 2x, \quad \int x dx = \frac{x^2}{2}, \quad \int x^2 dx = \frac{x^3}{3}.$$

5. Apply the bounds, taking care with negative signs.

$$\begin{aligned} A &= \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^1 \\ &= \left(2 - \frac{1}{2} - \frac{1}{3} \right) - \left(-4 - 2 + \frac{8}{3} \right) \\ &= \frac{7}{6} - \left(-\frac{10}{3} \right) = \boxed{\frac{9}{2}}. \end{aligned}$$