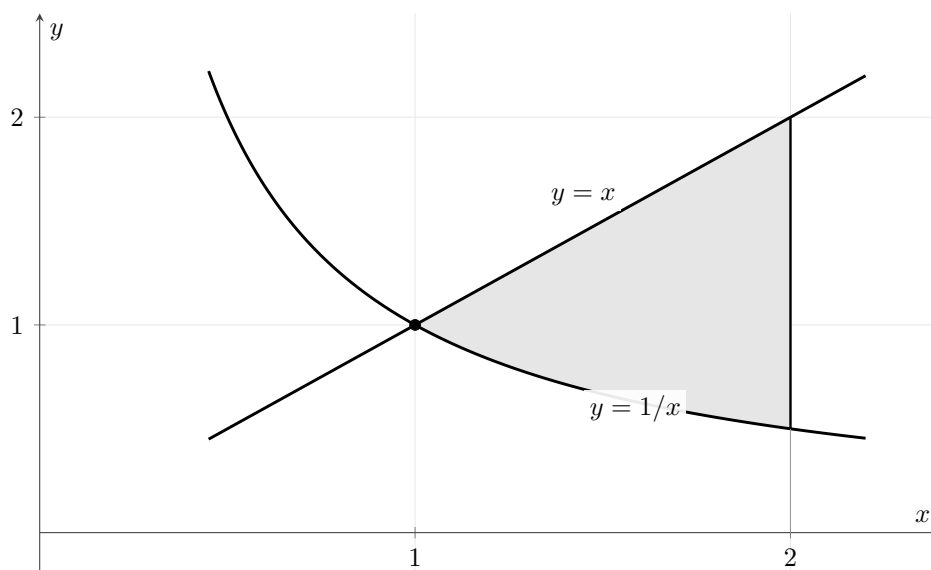


Question 7

Find the exact area of the region between the curves

$$y = \frac{1}{x} \quad \text{and} \quad y = x,$$

between $x = 1$ and $x = 2$.



The shaded region is the area to be found.

Question 7 – Solution

1. Identify the upper curve on $[1, 2]$.

For $x \geq 1$,

$$x - \frac{1}{x} = \frac{x^2 - 1}{x} \geq 0.$$

Thus the line $y = x$ lies above $y = 1/x$. They meet at $(1, 1)$, and the boundary $x = 2$ closes the region.

2. Set up the area.

$$A = \int_1^2 \left(x - \frac{1}{x} \right) dx.$$

3. Integrate each term.

Since $x > 0$ on this interval, an antiderivative of $1/x$ is $\ln x$.

$$\int x \, dx = \frac{x^2}{2}, \quad \int \frac{1}{x} \, dx = \ln x.$$

4. Substitute the bounds and simplify.

$$\begin{aligned} A &= \left[\frac{x^2}{2} - \ln x \right]_1^2 \\ &= (2 - \ln 2) - \left(\frac{1}{2} - \ln 1 \right) \\ &= \boxed{\frac{3}{2} - \ln 2}. \end{aligned}$$

5. Check the result.

Using $\ln 2 \approx 0.69315$, the area is $1.5 - 0.69315 \approx 0.80685$ square units. The result is positive, as required.