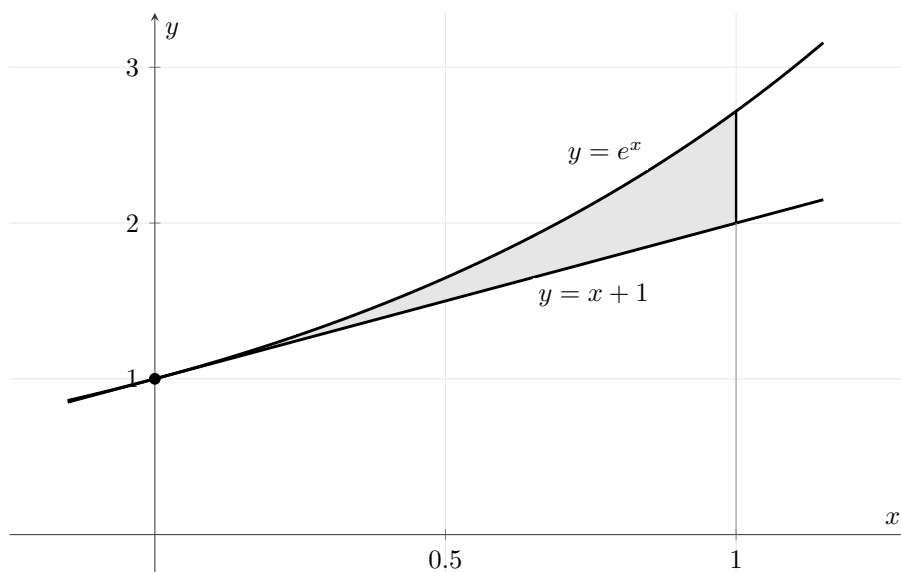


Question 6

Find the exact area of the region bounded by the curves

$$y = e^x \quad \text{and} \quad y = x + 1,$$

and the vertical lines $x = 0$ and $x = 1$.



The shaded region is the area to be found.

Question 6 – Solution

1. Identify the upper curve on $[0, 1]$.

Let $h(x) = e^x - x - 1$. Then $h(0) = 0$ and

$$h'(x) = e^x - 1 \geq 0 \quad (0 \leq x \leq 1).$$

Thus $e^x \geq x + 1$, with equality only at $x = 0$.

2. Use the given vertical boundaries.

$$A = \int_0^1 (e^x - (x + 1)) \, dx.$$

3. Integrate term by term.

$$\int e^x \, dx = e^x, \quad \int x \, dx = \frac{x^2}{2}, \quad \int 1 \, dx = x.$$

4. Evaluate at $x = 1$ and subtract the value at $x = 0$.

$$\begin{aligned} A &= \left[e^x - \frac{x^2}{2} - x \right]_0^1 \\ &= \left(e - \frac{1}{2} - 1 \right) - (1 - 0 - 0) \\ &= \boxed{e - \frac{5}{2}}. \end{aligned}$$

5. Check the sign and units.

The area is approximately $2.71828 - 2.5 = 0.21828$ square units. It is positive because e^x lies above the line on $(0, 1]$.